

الصف الثالث الإعدادي

الترم الثاني - خمسة

Prep. [ 3 ]

Second Term

1

Geometry

Sheet

Mr. Mahmoud Esamiel

01110882717

01006487539



# Prep [ 3 ] - Second Term - Geometry - Unit [ 4 ] - The Circle

## Lesson [ 1 ] : Basic Definitions And Concepts

### The circle

It is the set of points of the plane which are at a constant distance from a fixed point in the same plane.

- The fixed point is called "**the centre of the circle**".
- The constant distance is called "the radius length of the circle".
- The circle is usually denoted by its centre , so we say the circle M to mean the circle whose centre is the point M.

The centre of the circle



### Partition of the plane by the circle

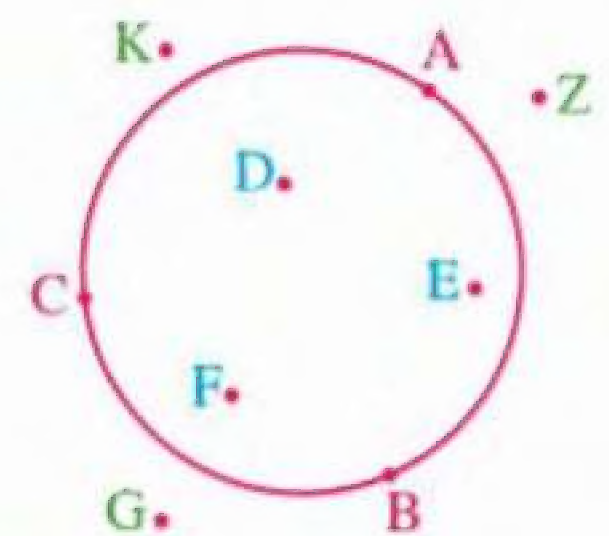
Any circle divides the plane into three sets of points which are :

- 1 The set of points of the circle.
- 2 The set of points inside the circle.
- 3 The set of points outside the circle.

For example :

The drawn circle in the opposite figure divides the plane into :

- 1 The set of points of the circle «on the circle» as : A , B , C , ...
- 2 The set of points inside the circle as : D , E , F , ...
- 3 The set of points outside the circle as : Z , K , G , ...



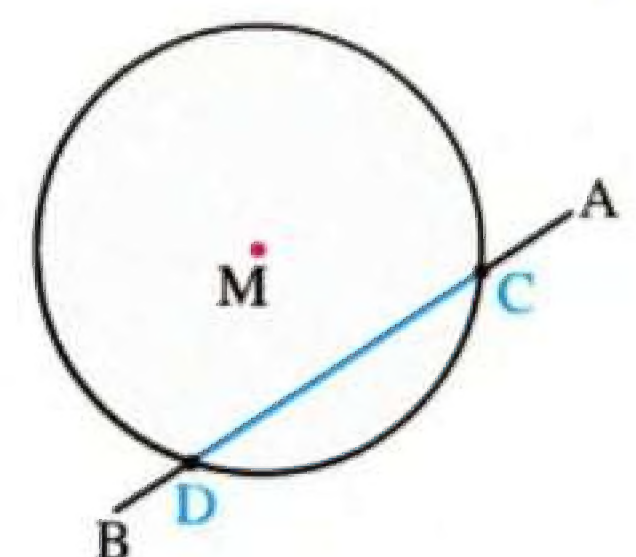
**The surface of the circle is :** the set of points of the circle  $\cup$  the set of points inside it.

So , the surface of the circle differs from the circle.

For example:

In the opposite figure :

- $\overline{AB} \cap \text{the circle} = \{C, D\}$  but  $\overline{AB} \cap \text{the surface of the circle} = \overline{CD}$
- $M \notin \text{the circle}$  but  $M \in \text{the surface of the circle}$ .





### The radius of the circle

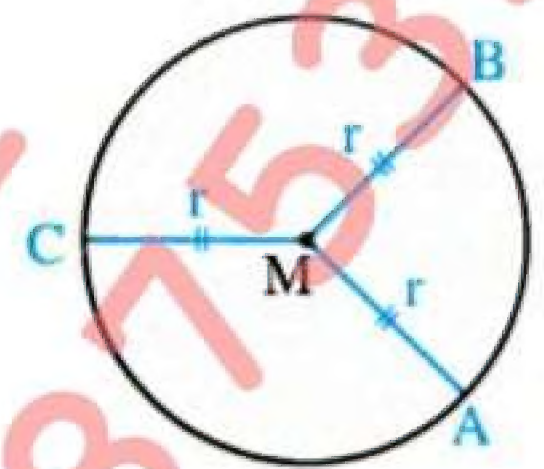
It is a line segment whose endpoints are the centre of the circle and any point on the circle.

In the opposite figure :

If the points A , B and C belong to the circle M ,

then  $\overline{MA}$  ,  $\overline{MB}$  and  $\overline{MC}$  are called radii of the circle M

and  $MA = MB = MC = r$  (where r is the radius length of the circle).



Notice that :

- 1 Any circle has an infinite number of radii and all of them are equal in length.
- 2 If two radii of two circles are equal in length , then the two circles are congruent and vice versa.

### The chord of the circle

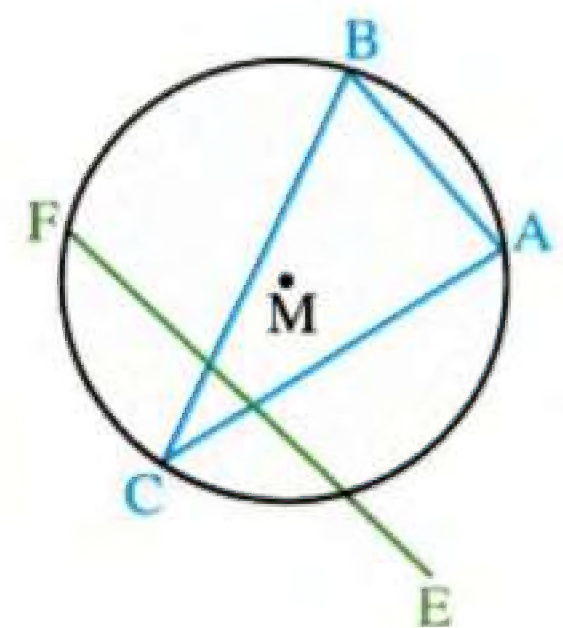
It is a line segment whose endpoints are any two points on the circle.

In the opposite figure :

If A , B and C belong to the circle M ,

then each of  $\overline{AB}$  ,  $\overline{AC}$  and  $\overline{BC}$

is a chord of the circle M



Notice that :

$\overline{EF}$  is not a chord of the circle M because  $E \notin$  the circle M

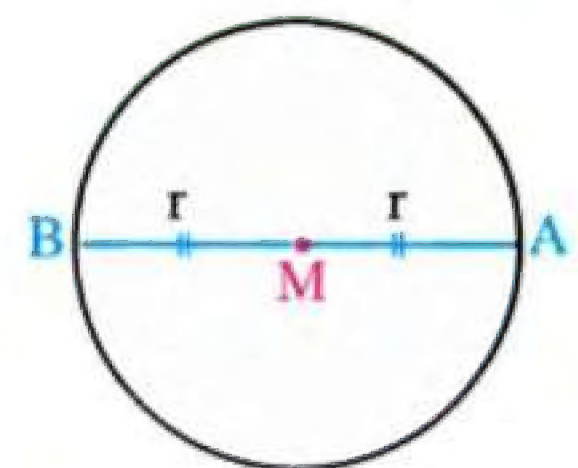
### The diameter of the circle

It is a chord passing through the centre of the circle.

In the opposite figure :

If M is a circle ,  $\overline{AB}$  is a chord of it

,  $M \in \overline{AB}$  , then  $\overline{AB}$  is a diameter of the circle M





**Notice that :**

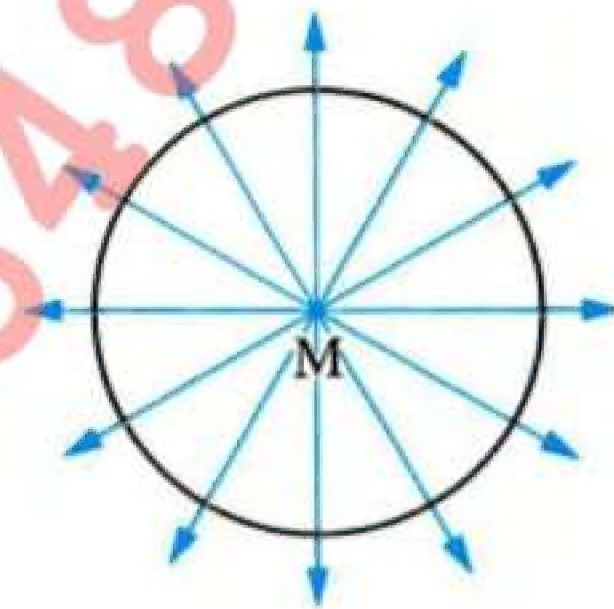
- 1 Any circle has an infinite number of diameters and all of them are equal in length.
- 2 The diameter of the circle is the longest chord of the circle , and its length =  $2r$

**The circumference of the circle and its area**

- The circumference of the circle =  $2\pi r$
- The area of the circle =  $\pi r^2$

**Symmetry in the circle**

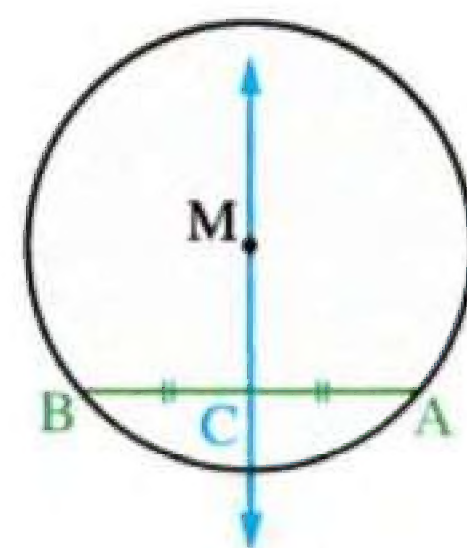
- Any straight line passing through the centre of the circle is an **axis of symmetry** of it.
- Since the number of these straight lines are infinite , then the circle has an infinite number of axes of symmetry.

**Important corollaries****Corollary 1**

The straight line passing through the **centre** of the circle and the **midpoint** of any chord of it is **perpendicular** to this chord.

**In the opposite figure :**

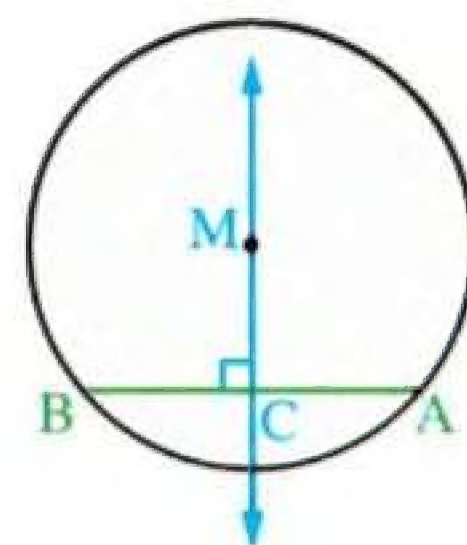
If  $\overline{AB}$  is a chord of the circle M  
and C is the midpoint of  $\overline{AB}$  , then  $\overrightarrow{MC} \perp \overline{AB}$

**Corollary 2**

The straight line passing through the **centre** of the circle and **perpendicular** to any chord of it **bisects** this chord.

**In the opposite figure :**

If  $\overline{AB}$  is a chord of the circle M and  $\overrightarrow{MC} \perp \overline{AB}$  , where  $C \in \overline{AB}$  , then C is the midpoint of  $\overline{AB}$





**Corollary 3**

The perpendicular bisector to any chord of a circle passes through the centre of the circle.

In the opposite figure :

If  $\overline{AB}$  is a chord of the circle  $M$  ,  $C$  is the midpoint of  $\overline{AB}$   
and the straight line  $L \perp \overline{AB}$  from the point  $C$  ,  
then  $M \in$  the straight line  $L$



From the previous, we deduce that :

The axis of symmetry of any chord of a circle passes through its centre , so this axis is also an axis of symmetry of the circle.

## Exercises

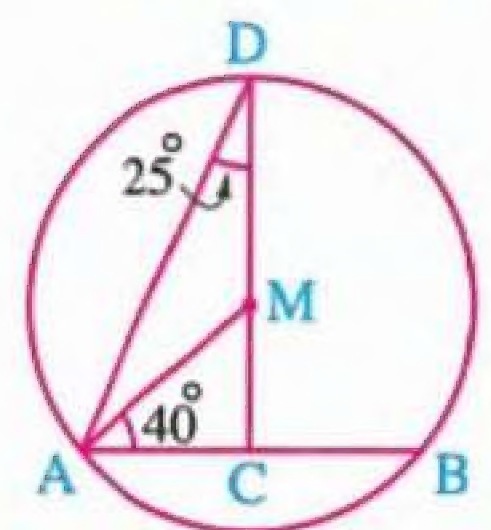
### [ A ] Essay problems : -

In the opposite figure :

$\overline{AB}$  is a chord of the circle  $M$  ,  
 $m(\angle D) = 25^\circ$   
and  $m(\angle MAC) = 40^\circ$

Prove that :

$C$  is the midpoint of  $\overline{AB}$



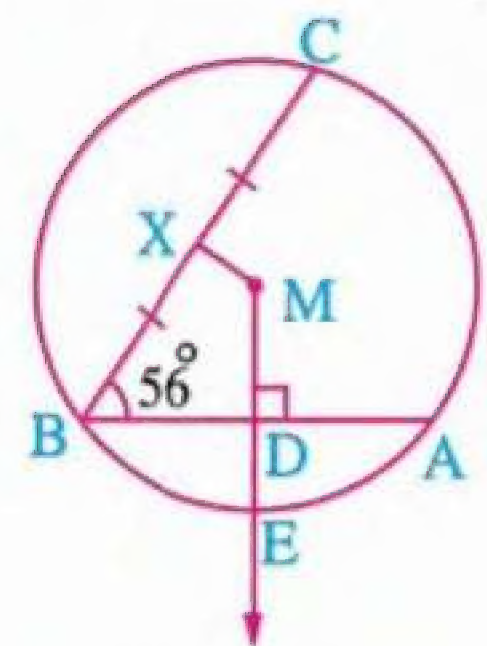
(Kafr El-Sheikh 09)

In the opposite figure :

$\overline{AB}$  and  $\overline{BC}$  are two chords in circle  $M$  ,  
which has radius length of 5 cm.,  
 $\overrightarrow{MD} \perp \overline{AB}$  intersects  $\overline{AB}$  at  $D$  and intersects the circle  $M$  at  $E$  ,  
 $X$  is the midpoint of  $\overline{BC}$  ,  $AB = 8$  cm.,  $m(\angle ABC) = 56^\circ$

Find : **1**  $m(\angle DMX)$

**2** The length of  $\overline{DE}$



(El-Menia 19 , El-Gharbia 17 , Souhag 15) «  $124^\circ$  , 2 cm. »



3

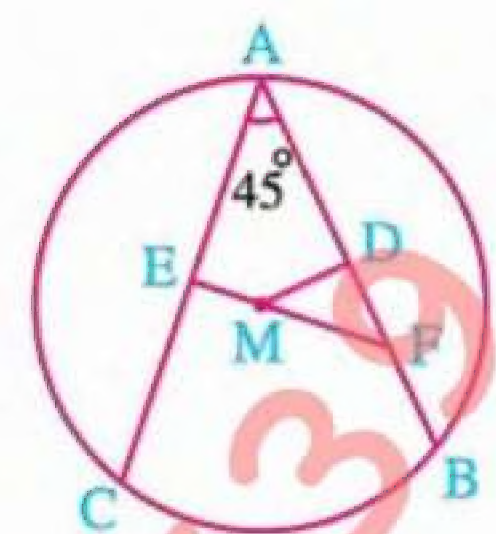
**In the opposite figure :**

$\overline{AB}$  and  $\overline{AC}$  are two chords of the circle M ,

$m(\angle BAC) = 45^\circ$  ,

D and E are the midpoints of  $\overline{AB}$  and  $\overline{AC}$  respectively.

**Prove that :**  $\triangle DFM$  is an isosceles triangle.



(New Valley 05)

4

**In the opposite figure :**

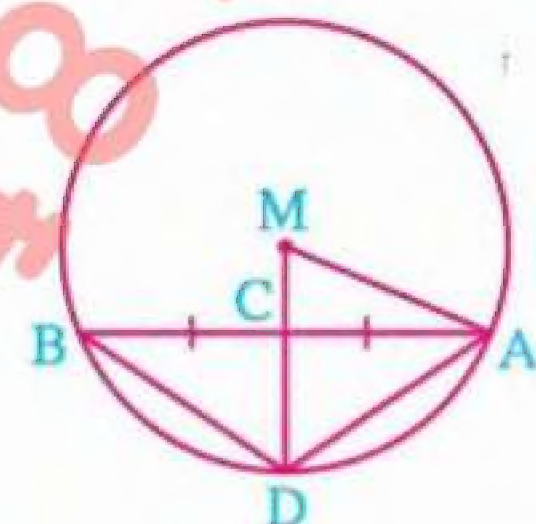
M is a circle of radius length 13 cm. ,

$\overline{AB}$  is a chord of length 24 cm. ,

C is the midpoint of  $\overline{AB}$

and  $\overline{MC} \cap \text{circle M} = \{D\}$

**Find :** The area of the triangle ADB



(El-Dakahlia 13) « 96 cm<sup>2</sup> »

5

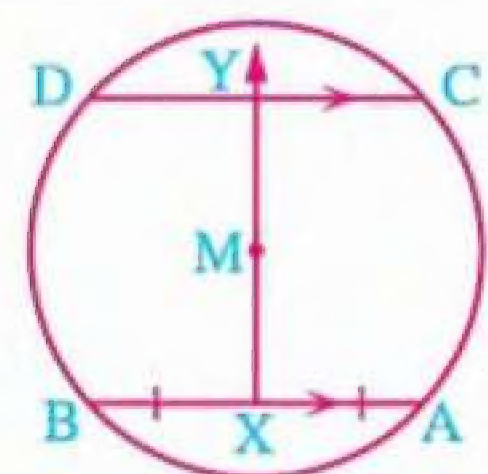
**In the opposite figure :**

M is a circle ,  $\overline{AB} \parallel \overline{CD}$  ,

X is the midpoint of  $\overline{AB}$

and  $\overline{XM}$  is drawn to cut  $\overline{CD}$  at Y

**Prove that :** Y is the midpoint of  $\overline{CD}$



(El-Menia 18 , Assiut 18 , Aswan 15 , Alexandria 13)

6

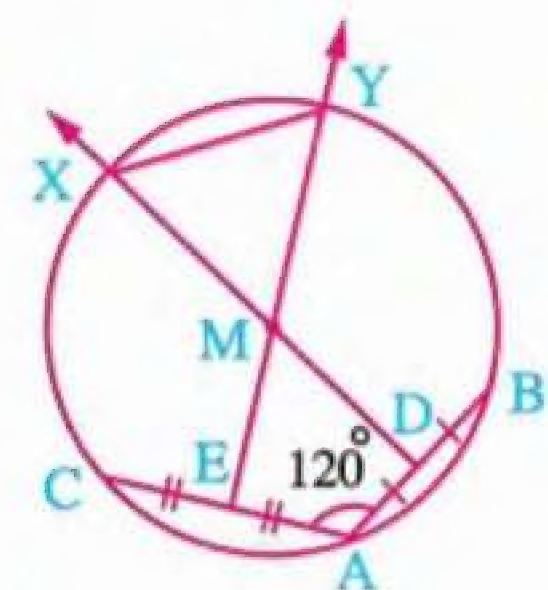
**In the opposite figure :**

$\overline{AB}$  and  $\overline{AC}$  are two chords in circle M

that includes an angle of measure  $120^\circ$  ,

D and E are the two midpoints of  $\overline{AB}$  and  $\overline{AC}$  respectively ,  $\overline{DM}$  and  $\overline{EM}$  are drawn to intersect the circle at X and Y respectively.

**Prove that :** The triangle XYM is an equilateral triangle.



(Aswan 16 , Beni Suef 15)

7

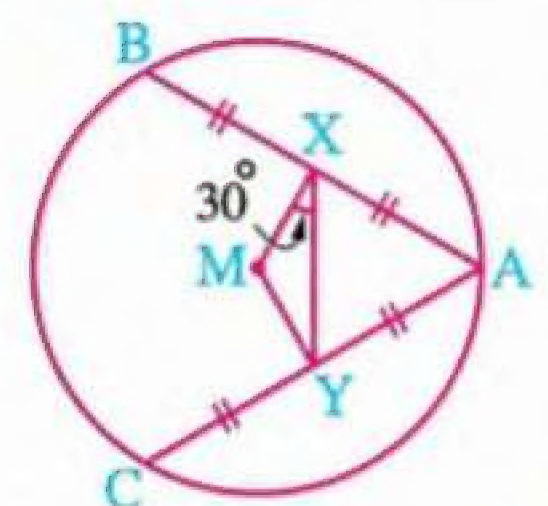
**In the opposite figure :**

$AC = AB$  , X is the midpoint of  $\overline{AB}$  ,

Y is the midpoint of  $\overline{AC}$  ,

$m(\angle MXY) = 30^\circ$

**Prove that :** The triangle AXY is equilateral.



(Assiut 14)



8

 In the opposite figure :


Two concentric circles with centre M ,  
 $\overline{AB}$  is a chord of the greater circle  
 and intersects the smaller circle at C , D  
 and  $\overline{ME} \perp \overline{AB}$

**Prove that :**  $AC = BD$

(El-Gharbia 18 , Qena 18 , Qena 17 , Red Sea 12)



9

 In the opposite figure :

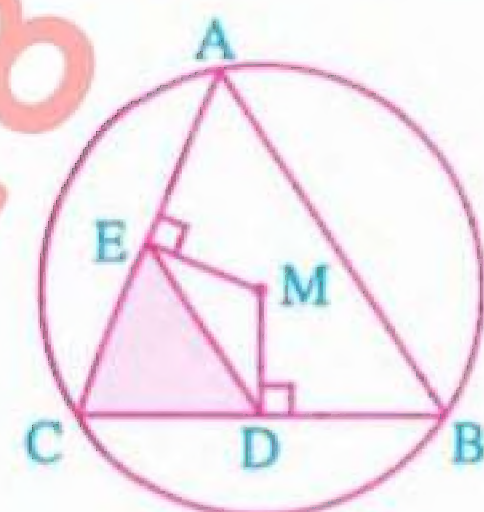
ABC is a triangle drawn inside a circle with centre M (inscribed triangle) ,  $\overline{MD} \perp \overline{BC}$  and  $\overline{ME} \perp \overline{AC}$

**Prove that :**

1  $\overline{ED} \parallel \overline{AB}$

2 The perimeter of  $\triangle CDE = \frac{1}{2}$  the perimeter of  $\triangle ABC$

(Kaf El-Sheikh 16 , El-Beheira 13)



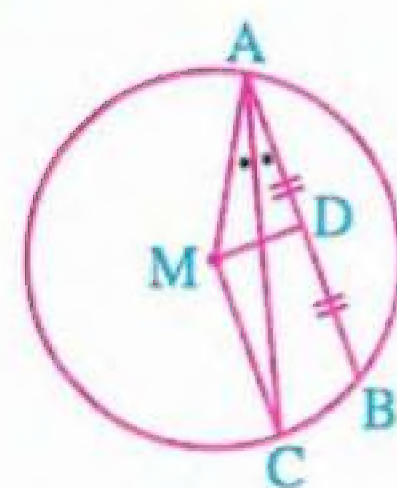
10

 In the opposite figure :

$\overline{AB}$  is a chord of circle M ,  
 $\overline{AC}$  bisects  $\angle BAM$  and intersects circle M at C  
 If D is the midpoint of  $\overline{AB}$

**Prove that :**  $\overline{DM} \perp \overline{CM}$

(El-Beheira 19 , El-Gharbia 17 , Souhag 14)

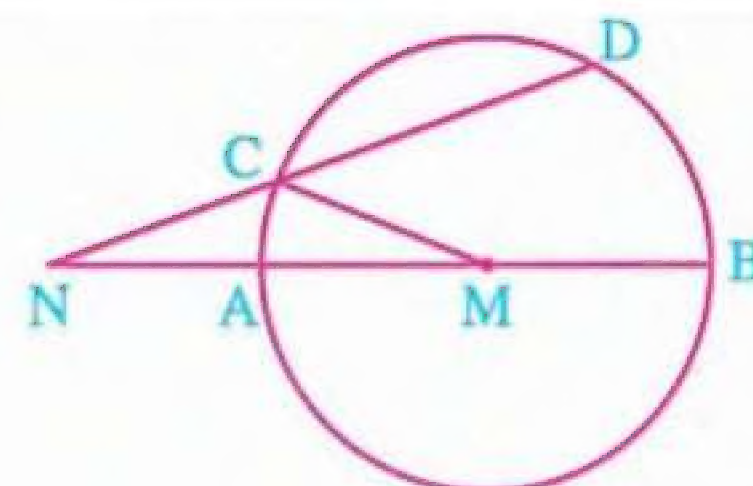


11

**In the opposite figure :**

$\overline{AB}$  is a diameter in circle M  
 $\overline{BA} \cap \overline{DC} = \{N\}$

**Prove that :**  $NC > NA$



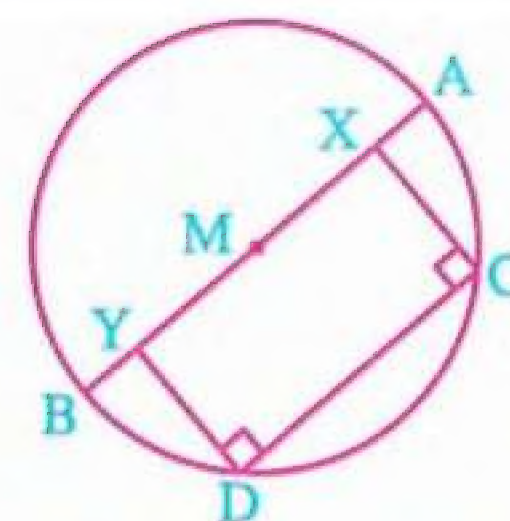
« El-Beheira 18 »

12

**In the opposite figure :**

$\overline{AB}$  is a diameter of the circle M ,  
 $\overline{CD}$  is a chord of it ,  $\overline{XC} \perp \overline{CD}$   
 and  $\overline{YD} \perp \overline{CD}$

**Prove that :**  $AX = BY$

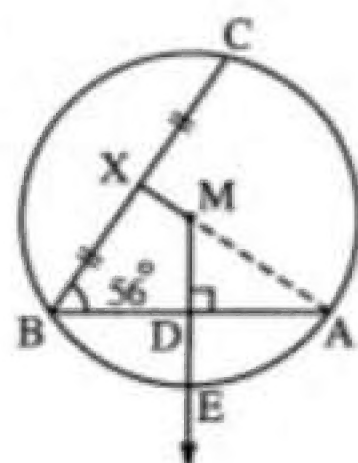


(Sharkia 09)



## Solutions

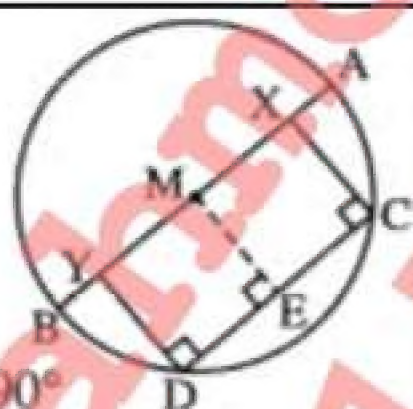
A	Essay Problems
1	$\because MA = MD = r$ $\therefore \Delta AMD$ is an isosceles triangle. $\therefore m(\angle DAM) = m(\angle ADM) = 25^\circ$ $\therefore m(\angle DAC) = 25^\circ + 40^\circ = 65^\circ$ $\therefore$ In $\Delta ADC$ : $m(\angle ACD) = 180^\circ - (25^\circ + 65^\circ) = 90^\circ$ $\therefore \overline{DC} \perp \overline{AB} \quad \because M \in \overline{DC}$ $\therefore C$ is the midpoint of $\overline{AB}$ (Q.E.D)
2	$\because X$ is the midpoint of $\overline{CB}$ $\therefore \overline{MX} \perp \overline{BC}$ $\therefore m(\angle DMX) = 360^\circ - (90^\circ + 90^\circ + 56^\circ) = 124^\circ$ (First req.) $\therefore \overline{MD} \perp \overline{AB}$ $\therefore D$ is the midpoint of $\overline{AB}$ $\therefore AD = 4$ cm. In $\Delta ADM$ : $\because m(\angle ADM) = 90^\circ, AM = r = 5$ cm. $\therefore MD = \sqrt{(AM)^2 - (AD)^2} = \sqrt{25 - 16} = \sqrt{9} = 3$ cm. $\therefore DE = 5 - 3 = 2$ cm. (Second req.)
3	$\because D$ is the midpoint of $\overline{AB}$ $\therefore \overline{MD} \perp \overline{AB}$ $\therefore m(\angle BDM) = 90^\circ$ similarly $m(\angle MEA) = 90^\circ$ $\therefore$ From $\Delta AFE$ : $m(\angle DFM) = 45^\circ$ and from $\Delta DFM$ : $m(\angle DMF) = 45^\circ$ $\therefore \Delta DFM$ is an isosceles triangle. (Q.E.D)
4	$\because C$ is the midpoint of $\overline{AB}$ $\therefore \overline{MD} \perp \overline{AB}$ In $\Delta ACM$ : $\because m(\angle ACM) = 90^\circ$ $\therefore (MC)^2 = (AM)^2 - (AC)^2$ (Pythagoras' theorem) $\therefore (MC)^2 = (13)^2 - (12)^2 = 25 \quad \therefore MC = 5$ cm. $\therefore CD = MD - MC = 13 - 5 = 8$ cm. $\therefore$ The area of $\Delta ADB = \frac{1}{2} \times 24 \times 8 = 96$ cm <sup>2</sup> (The req.)



5	$\because X$ is the midpoint of $\overline{AB}$ $\therefore \overline{MX} \perp \overline{AB} \quad \therefore m(\angle AXY) = 90^\circ$ $\because \overline{AB} \parallel \overline{CD}, \overline{XY}$ is a transversal $\therefore m(\angle XYD) = m(\angle AXY) = 90^\circ$ (alternate angles) $\therefore \overline{MY} \perp \overline{CD}$ $\therefore Y$ is the midpoint of $\overline{CD}$ (Q.E.D)
6	$\because D$ is the midpoint of $\overline{AB}$ $\therefore \overline{MD} \perp \overline{AB}$ $\because E$ is the midpoint of $\overline{AC} \quad \therefore \overline{ME} \perp \overline{AC}$ $\therefore m(\angle DME) = 360^\circ - (120^\circ + 90^\circ + 90^\circ) = 60^\circ$ $\therefore m(\angle XMY) = m(\angle DME) = 60^\circ$ (V.O.A) $\therefore MX = MY = r$ $\therefore \Delta XYM$ is an equilateral triangle. (Q.E.D)
7	$\because X$ is the midpoint of $\overline{AB}$ $\therefore \overline{MX} \perp \overline{AB}$ $\therefore m(\angle AXY) = 90^\circ - 30^\circ = 60^\circ$ $\because AB = AC \quad \therefore \frac{1}{2} AB = \frac{1}{2} AC$ $\therefore AX = AY \quad \therefore m(\angle AXY) = 60^\circ$ $\therefore \Delta AXY$ is an equilateral triangle. (Q.E.D.)
8	In the great circle : $\therefore \overline{ME} \perp \overline{AB} \quad \therefore E$ is the midpoint of $\overline{AB}$ $\therefore AE = EB$ (1) In the small circle : $\therefore \overline{ME} \perp \overline{CD} \quad \therefore E$ is the midpoint of $\overline{CD}$ $\therefore CE = ED$ (2) Subtracting (2) from (1) : $\therefore AE - CE = EB - ED$ $\therefore AC = BD$ (Q.E.D)
9	$\therefore \overline{MD} \perp \overline{BC} \quad \therefore D$ is the midpoint of $\overline{BC}$ $\therefore \overline{ME} \perp \overline{AC} \quad \therefore E$ is the midpoint of $\overline{AC}$ $\therefore$ In $\Delta ABC$ : $\therefore D$ and $E$ are the two midpoints of $\overline{BC}$ and $\overline{AC}$ respectively. $\therefore \overline{ED} \parallel \overline{AB}$ (Q.E.D 1)



	<p><math>\therefore D</math> is the midpoint of <math>\overline{BC}</math></p> <p><math>\therefore DC = \frac{1}{2} BC</math> (1)</p> <p><math>\therefore E</math> is the midpoint of <math>\overline{AC}</math></p> <p><math>\therefore EC = \frac{1}{2} AC</math> (2)</p> <p><math>\therefore D</math> and <math>E</math> are the two midpoints of <math>\overline{BC}</math> and <math>\overline{AC}</math> respectively.</p> <p><math>\therefore DE = \frac{1}{2} AB</math> (3)</p> <p>Adding (1), (2) and (3) :</p> <p><math>\therefore</math> The perimeter of <math>\triangle CDE</math></p> <p><math>= \frac{1}{2}</math> the perimeter of <math>\triangle ABC</math> (Q.E.D. 2)</p>
10	<p>In <math>\triangle AMC</math> :</p> <p><math>\therefore AM = MC = r \therefore m(\angle MAC) = m(\angle ACM)</math></p> <p><math>\therefore m(\angle BAC) = m(\angle MAC)</math></p> <p><math>\therefore m(\angle BAC) = m(\angle ACM)</math> and they are alternate angles</p> <p><math>\therefore \overline{AB} \parallel \overline{CM}</math></p> <p><math>\therefore D</math> is the midpoint of <math>\overline{AB} \therefore \overline{MD} \perp \overline{AB}</math></p> <p><math>\therefore \overline{AB} \parallel \overline{CM} \therefore \overline{DM} \perp \overline{CM}</math> (Q.E.D.)</p>
11	<p>In <math>\triangle MNC</math> : <math>\therefore NC + MC &gt; NM</math> (triangle inequality)</p> <p><math>\therefore MA = MC = r \therefore NM = AN + MA</math></p> <p><math>\therefore NC + MC &gt; AN + MA</math></p> <p><math>\therefore NC &gt; AN</math> (Q.E.D.)</p>
12	<p><b>Construction :</b></p> <p>Draw <math>\overline{ME} \perp \overline{CD}</math> to cut it at <math>E</math></p> <p><b>Proof :</b> <math>\therefore \overline{ME} \perp \overline{CD}</math></p> <p><math>\therefore E</math> is the midpoint of <math>\overline{CD}</math></p> <p><math>\therefore m(\angle XCE) = m(\angle MED) = 90^\circ</math></p> <p>but they are corresponding angles</p> <p><math>\therefore \overline{XC} \parallel \overline{ME}</math> similarly <math>\overline{ME} \parallel \overline{YD}</math></p> <p><math>\therefore \overline{XC} \parallel \overline{ME} \parallel \overline{YD}</math></p> <p><math>\therefore \overline{XY}</math> and <math>\overline{CD}</math> are two transversals to them</p> <p><math>\therefore CE = ED \therefore XM = MY</math></p> <p><math>\therefore AM = BM = r \therefore AM - XM = BM - MY</math></p> <p><math>\therefore AX = BY</math> (Q.E.D.)</p>





الصف الثالث الإعدادي  
الترم الثاني - خمسة

Prep. [ 3 ]

Second Term

2

Geometry

Sheet

Mr. Mahmoud Esamiel

01110882717

01006487539



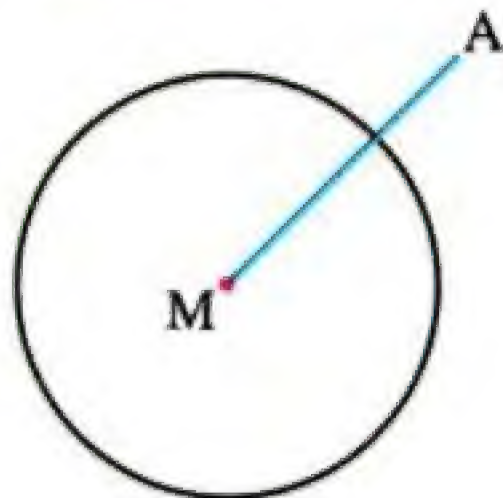
# Prep [ 3 ] - Second Term - Geometry - Unit [ 4 ] - The Circle

## Lesson [ 2 ] : Positions Of A Point and A Straight Line With Respect To A Circle

### First Position of a point with respect to a given circle

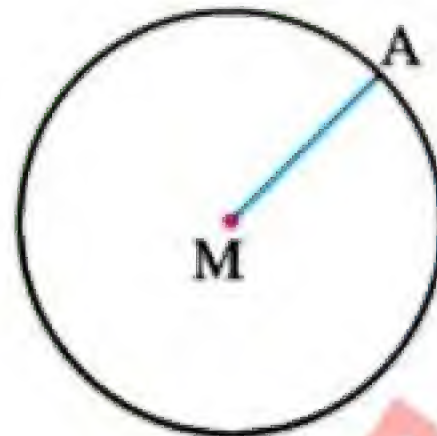
If  $M$  is a circle of radius length  $r$  and  $A$  is a point in its plane , then :

1  $A$  is **outside** the circle  $M$



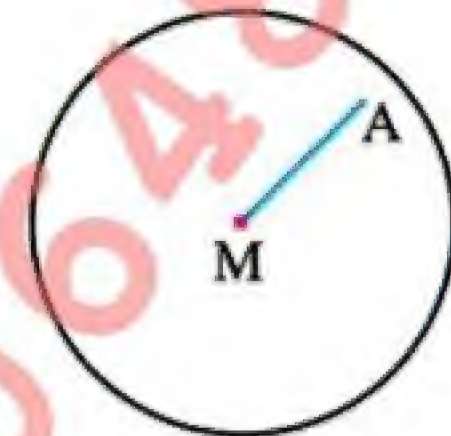
If  $MA > r$

2  $A$  is **on** the circle  $M$



If  $MA = r$

3  $A$  is **inside** the circle  $M$



If  $MA < r$

### Second Position of a straight line with respect to a circle

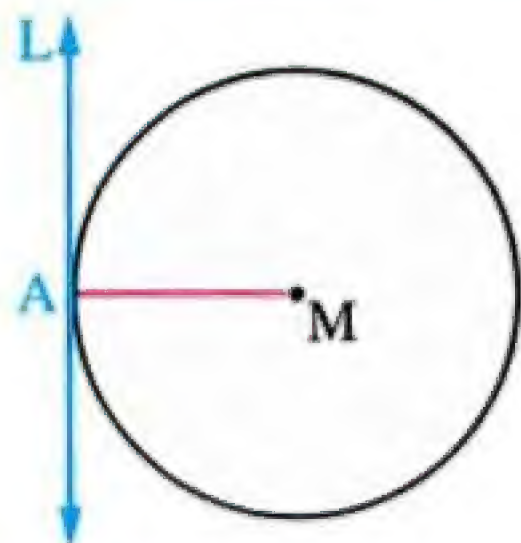
If	Then	The figure	Note that
1 $MA > r$	The straight line $L$ lies outside the circle $M$		<ul style="list-style-type: none"> <li><math>L \cap \text{the circle } M = \emptyset</math></li> <li><math>L \cap \text{the surface of the circle } M = \emptyset</math></li> </ul>
2 $MA = r$	The straight line $L$ is a tangent to the circle $M$ at $A$ . $A$ is called "the point of tangency"		<ul style="list-style-type: none"> <li><math>L \cap \text{the circle } M = \{A\}</math></li> <li><math>L \cap \text{the surface of the circle } M = \{A\}</math></li> </ul>
3 $MA < r$	The straight line $L$ is a secant to the circle $M$		<ul style="list-style-type: none"> <li><math>L \cap \text{the circle } M = \{X, Y\}</math></li> <li><math>L \cap \text{the surface of the circle } M = \overline{XY}</math></li> <li><math>\overline{XY}</math> is called the chord of intersection</li> </ul>

The two tangents which are drawn from the two endpoints of a diameter of a circle are parallel.



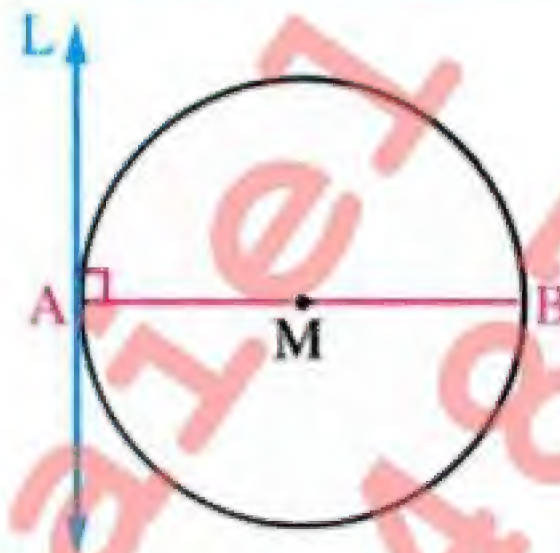
## Two important facts

- 1** The tangent to a circle is perpendicular to the radius drawn from the point of tangency.



i.e. if the straight line  $L$  is a tangent to the circle  $M$  at the point  $A$  ,  
then  $\overline{MA} \perp L$

- 2** The straight line which is perpendicular to the diameter of a circle at one of its endpoints is a tangent to the circle.



i.e. if  $\overline{AB}$  is a diameter of the circle  $M$  and the straight line  $L \perp \overline{AB}$  at the point  $A$  ,  
then  $L$  is a tangent to the circle  $M$  at the point  $A$

## Exercises

### [ A ] Essay problems : -

1

In the opposite figure :

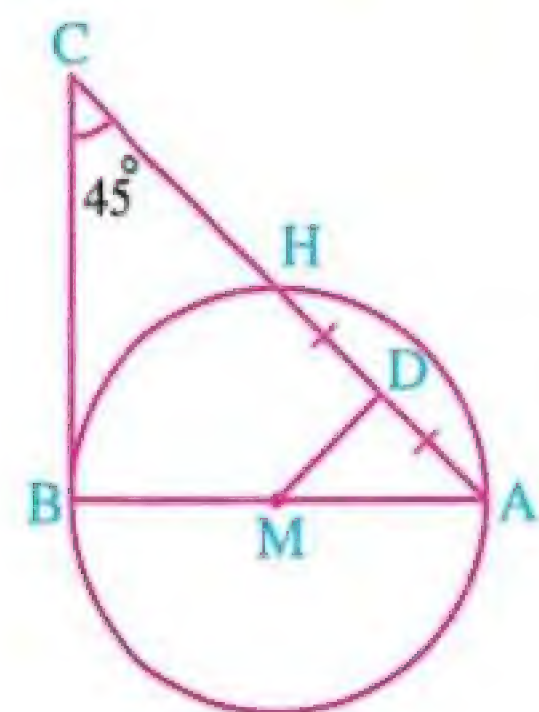
$\overline{BC}$  is a tangent at  $B$

,  $m(\angle C) = 45^\circ$

,  $D$  is the midpoint of  $\overline{AH}$

Prove that :  $DA = DM$

(Aswan II)



2

In the opposite figure :

$\overline{AB}$  is a diameter in the circle  $M$  ,

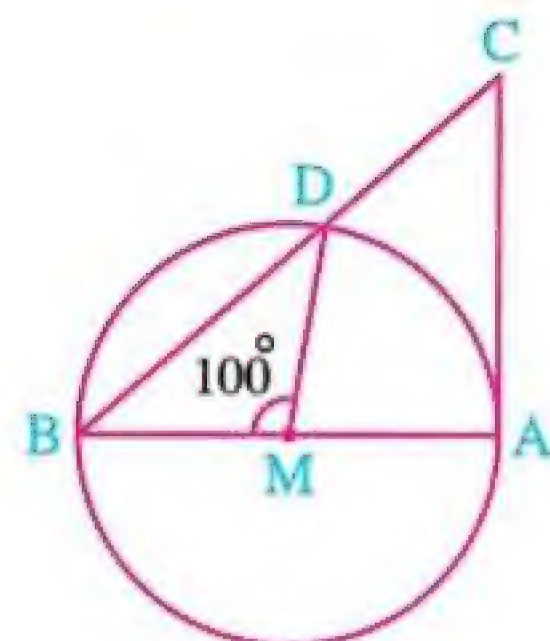
$\overline{AC}$  is a tangent to the circle at  $A$  ,

$m(\angle DMB) = 100^\circ$

Find by proof :

**1**  $m(\angle ACB)$

**2**  $m(\angle CDM)$



(El-Menia II) «  $50^\circ$  ,  $140^\circ$  »



3

 In the opposite figure :

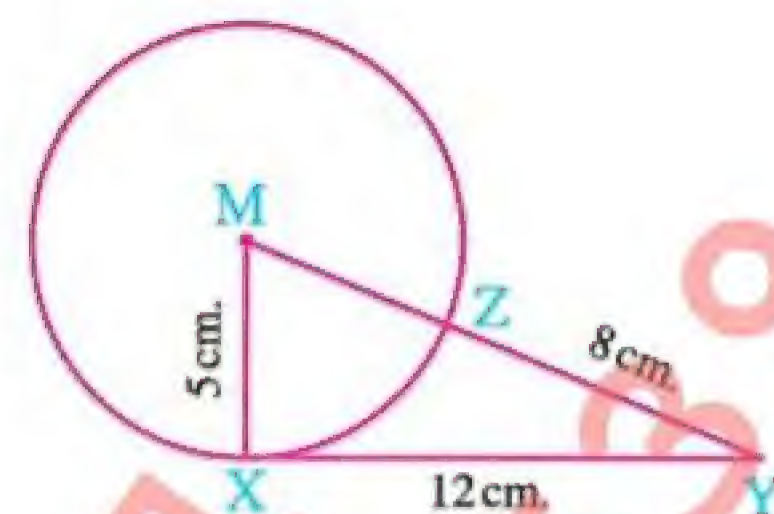
M is a circle with radius length 5 cm. ,

$XY = 12$  cm. ,  $\overline{MY} \cap \text{circle } M = \{Z\}$

and  $ZY = 8$  cm.

**Prove that :**  $\overleftrightarrow{XY}$  is a tangent to the circle M at X

(Matrouh 17 , South Sinai 16 , Qena 15 , El-Beheira 14)



4

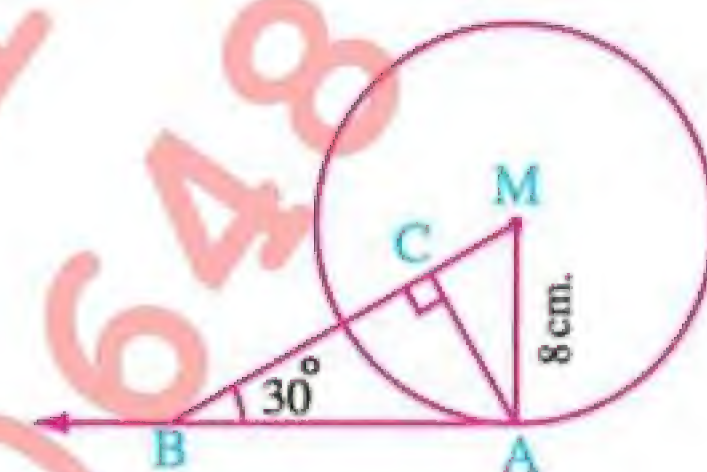
 In the opposite figure :

$\overleftrightarrow{AB}$  is a tangent to the circle M at A ,

$MA = 8$  cm. ,  $m(\angle ABM) = 30^\circ$  and  $\overline{AC} \perp \overline{MB}$

**Find :** The length of each of  $\overline{AB}$  and  $\overline{AC}$

(Giza 19 , Matrouh 18 , New Valley 18 , El-Monofia 14) «  $8\sqrt{3}$  cm. ,  $4\sqrt{3}$  cm. »



5

**In the opposite figure :**

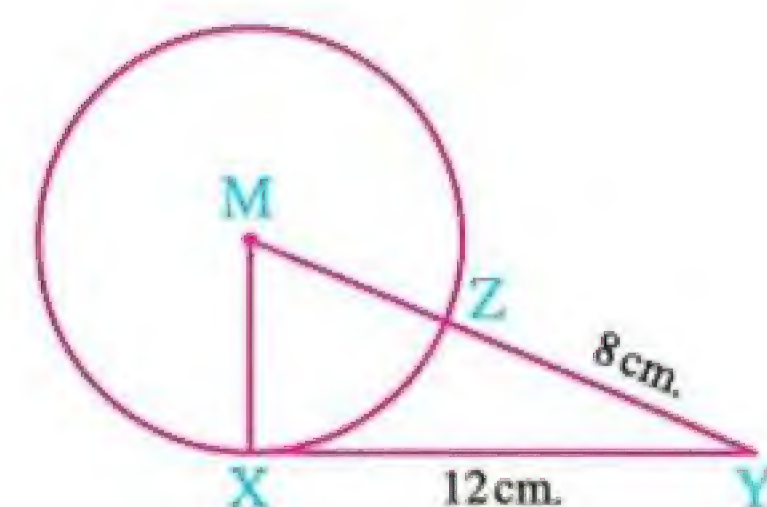
M is a circle ,  $\overleftrightarrow{XY}$  is a tangent to the circle at X

,  $\overline{MY} \cap \text{the circle } M = \{Z\}$  ,

$XY = 12$  cm. ,  $YZ = 8$  cm.

**Find :** The radius length of the circle.

(El-Menia 13) « 5 cm. »



6

**In the opposite figure :**

$\overleftrightarrow{AB}$  and  $\overleftrightarrow{AC}$  are two tangents to the circle M

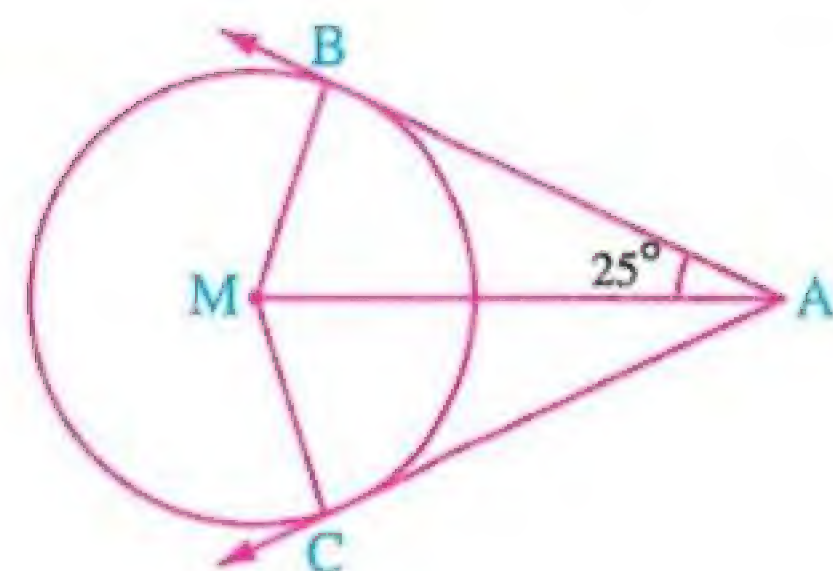
, touch it at B , C respectively

and  $m(\angle BAM) = 25^\circ$

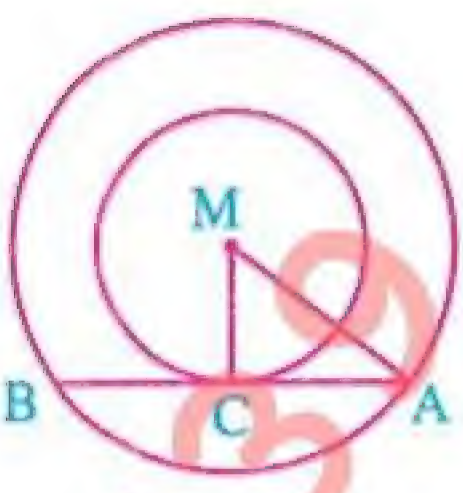
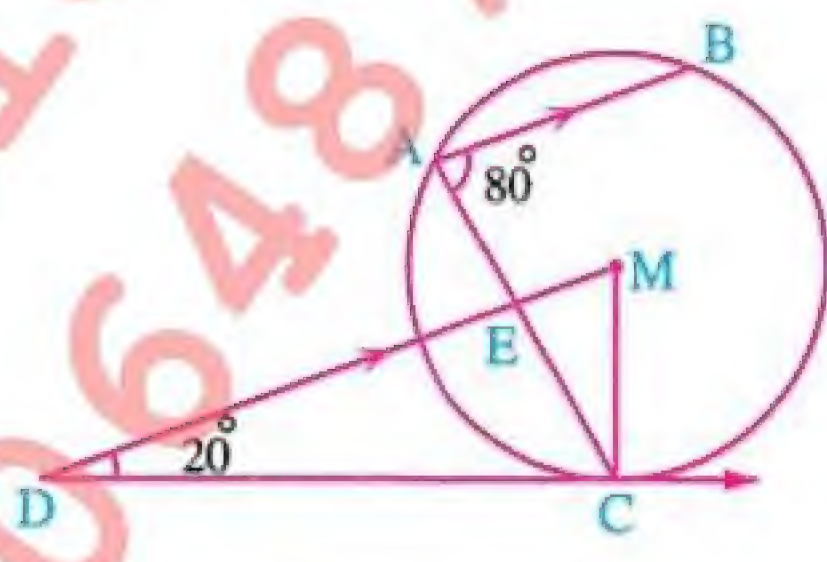

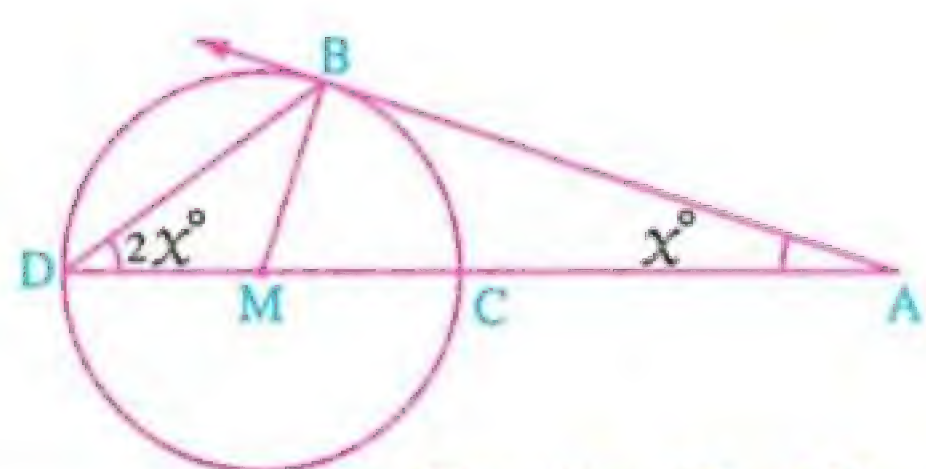
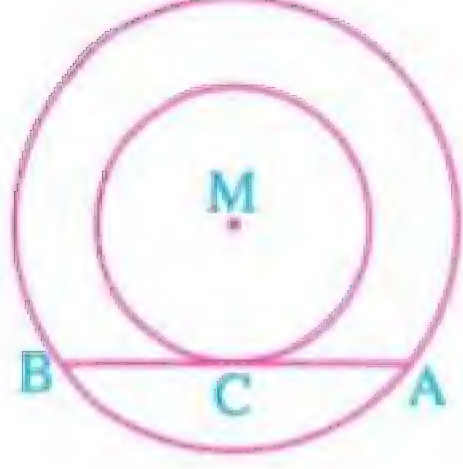
**1 Prove that :**  $\overleftrightarrow{MA}$  bisects  $\angle BMC$

**2 Find :**  $m(\angle BMC)$

(Port Said 17) «  $130^\circ$  »





7	<p><b>In the opposite figure :</b></p> <p><math>\overline{AB}</math> is a chord of the great circle and touches the small circle at C , <math>AB = 8</math> cm. and the radius length of the great circle = 5 cm.</p> <p><b>Find :</b> The radius length of the small circle.</p>	 <p>(Souhag 09) « 3 cm. »</p>
8	<p><b>In the opposite figure :</b></p> <p><math>\overrightarrow{DC}</math> touches the circle M at C , <math>\overline{AB} \parallel \overline{MD}</math> ,</p> <p><math>m(\angle BAC) = 80^\circ</math> , <math>m(\angle MDC) = 20^\circ</math></p> <p>and <math>\overline{AC} \cap \overline{MD} = \{E\}</math></p> <p><b>Find :</b> <math>m(\angle ECM)</math></p>	 <p>(Beni Suef 05) « <math>30^\circ</math> »</p>
9	<p><math>\overline{AB}</math> is a diameter in a circle of area <math>36\pi \text{ cm}^2</math> , <math>\overrightarrow{BC}</math> is drawn a tangent to the circle at B , if <math>m(\angle ACB) = 60^\circ</math> , then calculate the area of <math>\triangle ABC</math></p>	<p>(El-Dakahlia 14) « <math>24\sqrt{3} \text{ cm}^2</math> »</p>
10	<p><b>Prove that :</b> The points A (3 , - 1) , B (- 4 , 6) and C (2 , - 2) are located in circle whose centre is the point M (- 1 , 2) , then find the circumference of the circle.</p>	<p>(El-Beheira 11) « <math>10\pi</math> length units »</p>
11	<p> If <math>\overline{CD}</math> is a diameter of circle M where M (1 , 1) , D (3 , -2)</p> <p><b>Find :</b> The equation of the tangent to M at C</p>	<p>(El-Dakahlia 11) « <math>y = \frac{2}{3}x + 4\frac{2}{3}</math> »</p>
12	<p><b>In the opposite figure :</b></p> <p><math>\overrightarrow{AB}</math> touches the circle M at B , <math>\overline{CD}</math> is a diameter of it ,</p> <p><math>m(\angle BAM) = x^\circ</math> and <math>m(\angle MDB) = 2x^\circ</math></p> <p><b>Find :</b> The value of <math>x</math> in degrees.</p>	 <p>(Ismailia 06) « <math>18^\circ</math> »</p>
13	<p><b>In the opposite figure :</b></p> <p>Two circles are concentric at M</p> <p>, <math>\overline{AB}</math> is a chord in the greater circle and touches the smaller circle at C , if <math>AB = 14</math> cm.</p> <p><b>Find :</b> The area of the part included between the two circles.</p>	 <p>(El-Dakahlia 19) « <math>49\pi \text{ cm}^2</math> »</p>



14

In the opposite figure :

M and N are two congruent circles ,

$\overleftrightarrow{AB}$  is a common tangent to them ,

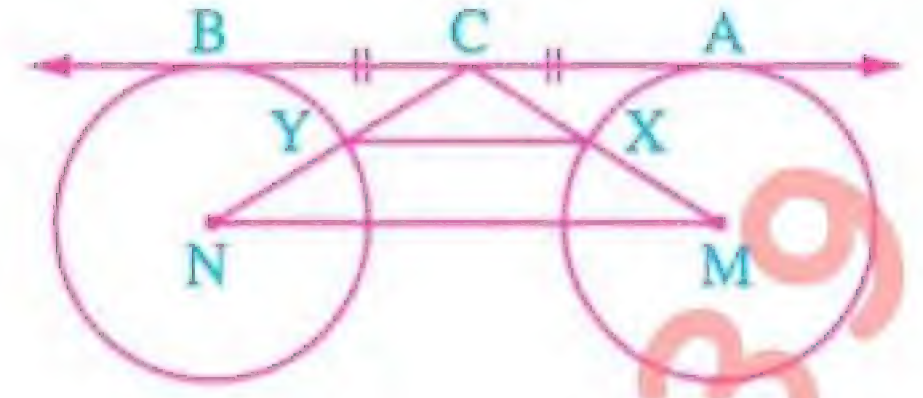
C is the midpoint of  $\overline{AB}$  ,

the circle  $M \cap \overline{MC} = \{X\}$  , the circle  $N \cap \overline{NC} = \{Y\}$

Prove that : 1  $\overline{AB} \parallel \overline{MN}$

2  $\triangle CMN$  is an isosceles triangle.



3  $\overline{XY} \parallel \overline{MN}$



(El-Kalyoubia 04)



**[ B ] Choose the correct : -**

1	<p>If a straight line L is a tangent to the circle M whose diameter length is 8 cm. , then L is at a distance of ..... cm. from its centre. <span style="float: right;">(Souhag 19 , El-Kalyoubia 18)</span></p> <p>(a) 3                      (b) 4                      (c) 6                      (d) 8</p>
2	<p>A circle M is of radius length 5 cm. , A is a point outside the circle , then MA equals ..... cm. <span style="float: right;">(Gharbia 03)</span></p> <p>(a) 3                      (b) 5                      (c) 8                      (d) 4</p>
3	<p>If the diameter length of a circle is 8 cm. and the straight line L is at distance of 3 cm. from its centre , then the straight line L is ..... <span style="float: right;">(Damietta 16 , El-Menia 14)</span></p> <p>(a) a tangent to the circle.                      (b) a secant to the circle. (c) outside the circle.                      (d) an axis of symmetry of the circle.</p>
4	<p>If M is a circle its diameter length = 14 cm. , <math>MA = (2x + 3)</math> cm. where A is a point on the circle , then <math>x =</math> ..... <span style="float: right;">(El-Kalyoubia 17 , El-Sharkia 15)</span></p> <p>(a) 5                      (b) 3                      (c) 2                      (d) 1</p>
5	<p> <math>\overline{AB}</math> is a diameter in a circle M , <math>\overleftrightarrow{AC}</math> and <math>\overleftrightarrow{BD}</math> are two tangents to the circle , then <math>\overleftrightarrow{AC}</math> ..... <math>\overleftrightarrow{BD}</math> <span style="float: right;">(Alexandria 13)</span></p> <p>(a) intersects                      (b) is perpendicular to (c) is parallel to                      (d) is coincident to</p>
6	<p> A circle is of a circumference <math>6\pi</math> cm. , and the straight line L is distant from its centre by 3 cm. , then the straight line L is ..... <span style="float: right;">(Red Sea 19 , Red Sea 17 , El-Monofia 15)</span></p> <p>(a) a tangent to the circle.                      (b) a secant. (c) outside the circle.                      (d) a diameter of the circle.</p>
7	<p>If the area of the circle M is <math>16\pi</math> cm<sup>2</sup> , A is a point in its plane where <math>MA = 8</math> cm. , then A lies ..... the circle M <span style="float: right;">(Qena 17 , El-Sharkia 09)</span></p> <p>(a) inside                      (b) outside                      (c) on                      (d) at the centre of</p>
8	<p>M is a circle with diameter of length 8 cm. If the straight line L is outside the circle , then the distance between the centre of the circle and the straight line L <math>\in</math> ..... <span style="float: right;">(Kaf El-Sheikh 14)</span></p> <p>(a) <math>]4 , \infty[</math>                      (b) <math>[0 , 4[</math>                      (c) <math>]0 , 4[</math>                      (d) <math>[0 , 8]</math></p>



9

A circle with diameter length  $(2x + 5)$  cm. , the straight line L is at a distance  $(x + 2)$  cm. from its centre , then the straight line L is .....

(Port Said 17)

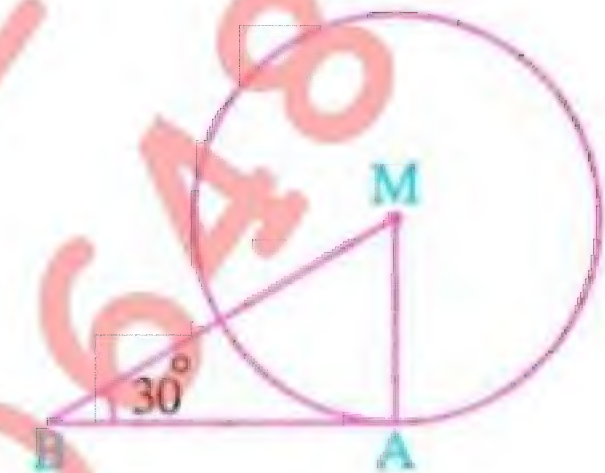
- (a) a secant to the circle at the two points.
- (b) outside the circle.
- (c) a tangent to the circle.
- (d) an axis of symmetry of the circle.

10

$\overrightarrow{AB}$  is a tangent to the circle M  
 ,  $m(\angle B) = 30^\circ$  ,  $AM = 6$  cm.  
 , then  $MB = \dots\dots\dots$  cm.

(Red Sea 18)

- (a) 3
- (b) 6
- (c) 9
- (d) 12



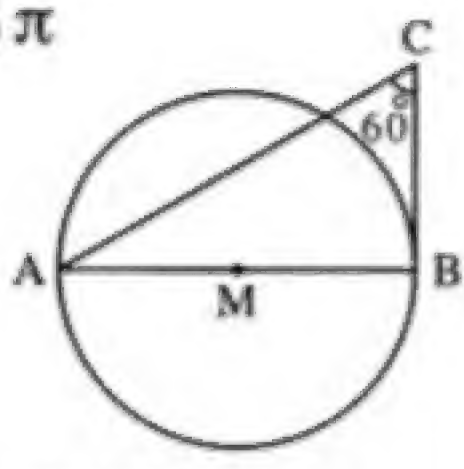


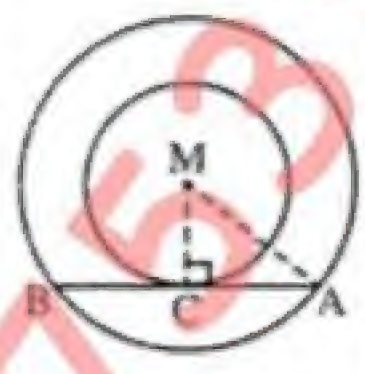
## Solutions

A	Essay Problems
1	<p><math>\therefore \overline{BC}</math> is a tangent to the circle M at B  <math>\therefore \overline{BC} \perp \overline{MB}</math>  In <math>\triangle ABC</math> : <math>m(\angle A) = 180^\circ - (45^\circ + 90^\circ) = 45^\circ</math>  <math>\therefore D</math> is the midpoint of <math>\overline{AH}</math> <math>\therefore \overline{MD} \perp \overline{AH}</math>  In <math>\triangle ADM</math> :  <math>m(\angle DMA) = 180^\circ - (45^\circ + 90^\circ) = 45^\circ</math>  <math>\therefore m(\angle DAM) = m(\angle DMA)</math>  <math>\therefore DA = DM</math> (Q.E.D.)</p>
2	<p>In <math>\triangle MDB</math> :  <math>\therefore MD = MB = r</math>  <math>\therefore m(\angle MBD) = m(\angle MDB) = \frac{180^\circ - 100^\circ}{2} = 40^\circ</math>  <math>\therefore \overline{AC}</math> is a tangent to the circle M at A  <math>\therefore \overline{MA} \perp \overline{AC}</math>  In <math>\triangle ABC</math> :  <math>m(\angle C) = 180^\circ - (90^\circ + 40^\circ) = 50^\circ</math> (First req.)  <math>m(\angle CDM) = 180^\circ - 40^\circ = 140^\circ</math> (Second req.)</p>
3	<p><math>\therefore MZ = r = 5</math> cm. <math>\therefore MY = 13</math> cm.  <math>\therefore (MY)^2 = 169</math> , <math>(MX)^2 = 25</math>  <math>\therefore (XY)^2 = 144</math>  <math>\therefore (MX)^2 + (XY)^2 = (MY)^2</math>  <math>\therefore m(\angle MXY) = 90^\circ</math> <math>\therefore \overline{XY} \perp \overline{MX}</math>  <math>\therefore \overline{XY}</math> is a tangent to the circle M at X (Q.E.D.)</p>
4	<p><math>\therefore \overline{AB}</math> is a tangent to the circle M at A  <math>\therefore m(\angle MAB) = 90^\circ</math>  <math>\therefore \tan(\angle B) = \frac{MA}{AB}</math> <math>\therefore \tan 30^\circ = \frac{8}{AB}</math>  <math>\therefore \frac{1}{\sqrt{3}} = \frac{8}{AB}</math> <math>\therefore AB = 8\sqrt{3}</math> cm.  In <math>\triangle ABC</math> which is right-angled at C  <math>\therefore m(\angle ABC) = 30^\circ</math>  <math>\therefore AC = \frac{1}{2} AB = \frac{1}{2} \times 8\sqrt{3}</math>  <math>= 4\sqrt{3}</math> cm. (Second req.)</p>

5	<p><math>\therefore \overline{XY}</math> is a tangent to the circle at X  <math>\therefore \overline{MX} \perp \overline{XY}</math> <math>\therefore m(\angle MXY) = 90^\circ</math>  <math>\therefore</math> In <math>\triangle MXY</math> : <math>(MY)^2 = (MX)^2 + (XY)^2</math>  <math>\therefore (MZ + 8)^2 = (MX)^2 + 144</math>  <math>\therefore MZ = MX = r</math> <math>\therefore (r + 8)^2 = r^2 + 144</math>  <math>\therefore r^2 + 16r + 64 = r^2 + 144</math> <math>\therefore 16r = 80</math>  <math>\therefore r = \frac{80}{16} = 5</math> cm. (The req.)</p>
6	<p><math>\therefore \overline{AB}</math> is a tangent to the circle M at B  <math>\therefore \overline{MB} \perp \overline{AB}</math> <math>\therefore m(\angle ABM) = 90^\circ</math>  <math>\therefore \overline{AC}</math> is a tangent to the circle M at C  <math>\therefore \overline{MC} \perp \overline{AC}</math> <math>\therefore m(\angle ACM) = 90^\circ</math>  <math>\therefore</math> In <math>\triangle ABM</math> , <math>\triangle ACM</math> which are right-angled  <math>\begin{cases} \overline{MB} = \overline{MC} = r \\ \overline{AM} \text{ is a common hypotenuse} \end{cases}</math>  <math>\therefore \triangle ABM \cong \triangle ACM</math>  <math>\therefore m(\angle AMB) = m(\angle AMC)</math>  <math>\therefore \overline{MA}</math> bisects <math>\angle BMC</math> (First req.)  From <math>\triangle ABM</math> : <math>m(\angle AMB) = 180^\circ - (90^\circ + 25^\circ) = 65^\circ</math>  <math>\therefore m(\angle BMC) = 2 \times 65^\circ = 130^\circ</math> (Second req.)</p>
7	<p>In the small circle :  <math>\therefore \overline{AB}</math> is a tangent at C <math>\therefore \overline{MC} \perp \overline{AB}</math>  In the great circle : <math>\therefore \overline{MC} \perp \overline{AB}</math>  <math>\therefore C</math> is the midpoint of <math>\overline{AB}</math> <math>\therefore AC = 4</math> cm.  <math>\therefore AM = 5</math> cm.  <math>\therefore</math> In <math>\triangle ACM</math> which is right-angled at C  <math>MC = \sqrt{(AM)^2 - (AC)^2} = \sqrt{25 - 16} = 3</math> cm. (The req.)</p>
8	<p><math>\therefore \overline{DC}</math> is a tangent to the circle M at C  <math>\therefore \overline{MC} \perp \overline{DC}</math> <math>\therefore m(\angle MCD) = 90^\circ</math>  <math>\therefore</math> In <math>\triangle DMC</math> : <math>m(\angle DMC) = 180^\circ - (90^\circ + 20^\circ) = 70^\circ</math>  <math>\therefore \overline{AB} \parallel \overline{MD}</math> , <math>\overline{AE}</math> is a transversal to them  <math>\therefore m(\angle MEC) = m(\angle BAE) = 80^\circ</math>  (corresponding angles)  <math>\therefore</math> In <math>\triangle MEC</math> :  <math>m(\angle ECM) = 180^\circ - (70^\circ + 80^\circ) = 30^\circ</math> (The req.)</p>



9	<p> <math>\therefore</math> The area of the circle <math>= 36 \pi</math>  <math>\therefore r^2 \pi = 36 \pi</math>  <math>\therefore r^2 = 36</math>  <math>\therefore r = 6 \text{ cm.}</math>  <math>\therefore AB = 12 \text{ cm.}</math>  <math>\therefore \overline{BC}</math> is a tangent to the circle M at B  <math>\therefore \overline{BC} \perp \overline{AB}</math>  In <math>\triangle ABC : \tan (\angle C) = \frac{AB}{BC} \quad \therefore \tan 60^\circ = \frac{12}{BC}</math>  <math>\therefore BC = \frac{12}{\tan (60^\circ)} = \frac{12}{\sqrt{3}} = 4\sqrt{3} \text{ cm.}</math>  <math>\therefore</math> The area of <math>\triangle ABC</math>  <math>= \frac{1}{2} AB \times BC = \frac{1}{2} \times 12 \times 4\sqrt{3}</math>  <math>= 24\sqrt{3} \text{ cm}^2 \quad (\text{The req.})</math> </p> 
10	<p> <math>\therefore MA = \sqrt{(-1-3)^2 + (2+1)^2} = \sqrt{16+9} = \sqrt{25}</math>  <math>= 5 \text{ length units}</math>  <math>\therefore MB = \sqrt{(-1+4)^2 + (2-6)^2} = \sqrt{9+16} = \sqrt{25}</math>  <math>= 5 \text{ length units}</math>  <math>\therefore MC = \sqrt{(-1-2)^2 + (2+2)^2} = \sqrt{9+16} = \sqrt{25}</math>  <math>= 5 \text{ length units}</math>  <math>\therefore MA = MB = MC</math>  <math>\therefore</math> The points A , B and C lie on the circle M (Q.E.D.1)  <math>\therefore</math> its circumference <math>= 10 \pi</math> length units. (Q.E.D.2) </p>
11	<p> <math>\therefore \overline{CD}</math> is a diameter in the circle M  <math>\therefore M</math> is the midpoint of <math>\overline{CD}</math>  Let <math>C (X , y) \quad \therefore (1 , 1) = \left( \frac{X+3}{2} , \frac{y-2}{2} \right)</math>  <math>\therefore \frac{X+3}{2} = 1 \quad \therefore X+3 = 2 \quad \therefore X = -1</math>  <math>\therefore \frac{y-2}{2} = 1 \quad \therefore y-2 = 2 \quad \therefore y = 4</math>  <math>\therefore C = (-1 , 4)</math>  <math>\therefore</math> the slope of <math>\overline{CD} = \frac{-2-4}{3+1} = \frac{-6}{4} = -\frac{3}{2}</math>  <math>\therefore</math> The slope of the perpendicular straight line to <math>\overline{CD} = \frac{2}{3}</math>  <math>\therefore</math> the tangent to the circle M at C is perpendicular to <math>\overline{CD}</math>  <math>\therefore</math> The slope of the tangent to the circle at C <math>= \frac{2}{3}</math>  <math>\therefore</math> The equation of the tangent is : <math>y = \frac{2}{3} X + c</math>  <math>\therefore</math> the tangent passes through the point C <math>(-1 , 4)</math> </p>

	<p> <math>\therefore 4 = \frac{2}{3} \times -1 + c \quad \therefore c = 4 \frac{2}{3}</math>  <math>\therefore</math> The equation is : <math>y = \frac{2}{3} X + 4 \frac{2}{3} \quad (\text{The req.})</math> </p>
12	<p> <math>\therefore \overline{AB}</math> touches the small circle at C  <math>\therefore \overline{MC} \perp \overline{AB}</math>  <math>\therefore \overline{AB}</math> is a chord of the great circle , <math>\overline{MC} \perp \overline{AB}</math>  <math>\therefore C</math> is the midpoint of <math>\overline{AB}</math>  <math>\therefore AC = \frac{14}{2} = 7 \text{ cm.}</math>  <math>\therefore \triangle AMC</math> is right angled at C  <math>\therefore (AC)^2 = (MA)^2 - (MC)^2</math>  <math>\therefore (7)^2 = (MA)^2 - (MC)^2</math>  <math>\therefore (MA)^2 - (MC)^2 = 49</math>  <math>\therefore</math> The area of the included part between the two circles = the area of the greater circle - the area of the smaller circle <math>= \pi (MA)^2 - \pi (MC)^2</math>  <math>= \pi [(MA)^2 - (MC)^2] = 49 \pi \text{ cm}^2 \quad (\text{The req.})</math> </p> 
13	<p> <math>\therefore \overline{AB}</math> is a tangent to the circle M at B  <math>\therefore \overline{MB} \perp \overline{AB} \quad \therefore m(\angle ABM) = 90^\circ</math>  <math>\therefore MB = MD</math> (lengths of two radii)  <math>\therefore m(\angle MBD) = m(\angle MDB) = 2 X^\circ</math>  In <math>\triangle ABD : m(\angle A) + m(\angle ABD) + m(\angle D) = 180^\circ</math>  <math>\therefore X^\circ + 90^\circ + 2 X^\circ + 2 X^\circ = 180^\circ</math>  <math>\therefore 5 X^\circ = 90^\circ \quad \therefore X = 18^\circ \quad (\text{The req.})</math> </p>



**Construction :** We draw  $\overline{MA}$  and  $\overline{NB}$

**Proof :**  $\because \overline{AB}$  is a tangent to the circle M at A

$$\therefore \overline{MA} \perp \overline{AB}$$

similarly  $\overline{NB} \perp \overline{AB}$

$$\therefore \overline{MA} \parallel \overline{NB}$$

$\because MA = NB$  (two radii of two congruent circles)

$\therefore AMNB$  is a parallelogram

$$\therefore \overline{AB} \parallel \overline{MN} \quad (\text{Q.E.D. 1})$$

In  $\triangle AMC \cong \triangle BNC$  :

$$\begin{cases} AC = BC & (\text{given}) \\ MA = BN & (\text{given}) \\ m(\angle MAC) = m(\angle NBC) = 90^\circ & (\text{proved}) \end{cases}$$

$$\therefore \triangle AMC \cong \triangle BNC \quad \therefore MC = NC \quad (1)$$

$\therefore \triangle CMN$  is an isosceles triangle (Q.E.D. 2)

$$\therefore MX = NY \quad (2)$$

$\therefore$  Subtracting (2) from (1) :

$$\therefore MC - MX = NC - NY \quad \therefore CX = CY$$

$\therefore$  From the isosceles triangle  $XCX$

$$\therefore m(\angle CXY) = \frac{180^\circ - m(\angle 1)}{2} \quad (3)$$

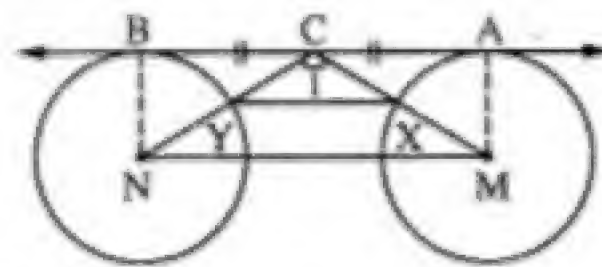
and from the isosceles triangle  $MNC$

$$\therefore m(\angle CMN) = \frac{180^\circ - m(\angle 1)}{2} \quad (4)$$

From (3) and (4) :  $m(\angle CXY) = m(\angle CMN)$

and they are corresponding angles

$$\therefore \overline{XY} \parallel \overline{MN} \quad (\text{Q.E.D. 3})$$



8

A

9

A

10

D

14

B

Choose

1

B

2

C

3

B

4

C

5

C

6

A

7

B



الصف الثالث الإعدادي

الترم الثاني - هندسة

Prep. [ 3 ]

Second Term

3

Geometry

Sheet

Mr. Mahmoud Esamiel

01110882717

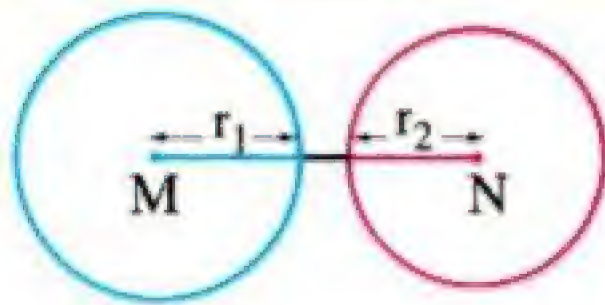
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# Prep [ 3 ] - Second Term - Geometry - Unit [ 4 ] - The Circle

## Lesson [ 3 ] : Positions Of A Circle With Respect To Another Circle

If  $MN > r_1 + r_2$

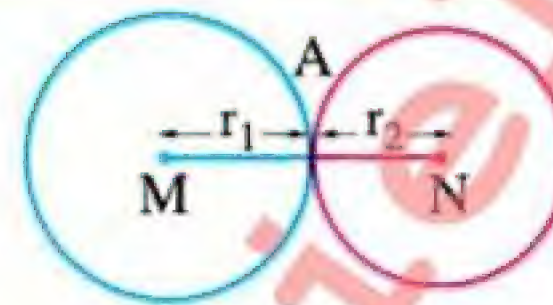


Then the two circles are : Distant

Notice that :

- The circle  $M \cap$  the circle  $N = \emptyset$
- The surface of circle  $M \cap$  the surface of circle  $N = \emptyset$

If  $MN = r_1 + r_2$

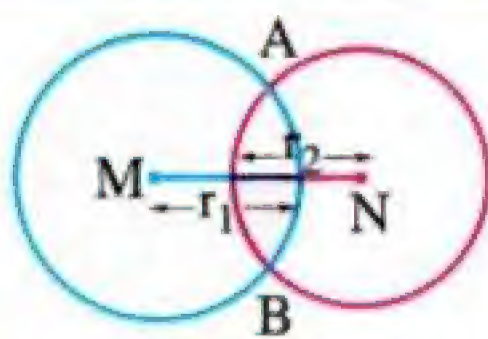


Then the two circles are :  
Touching externally

Notice that :

- The circle  $M \cap$  the circle  $N = \{A\}$
- The surface of circle  $M \cap$  the surface of circle  $N = \{A\}$

If  $r_1 - r_2 < MN < r_1 + r_2$

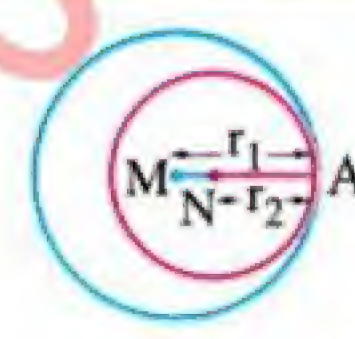


Then the two circles are :  
Intersecting

Notice that :

- The circle  $M \cap$  the circle  $N = \{A, B\}$
- The surface of circle  $M \cap$  the surface of circle  $N =$  the surface of the shaded part.

If  $MN = r_1 - r_2$

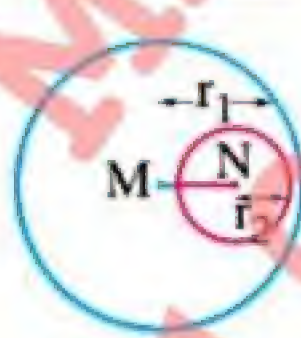


Then the two circles are :  
Touching internally

Notice that :

- The circle  $M \cap$  the circle  $N = \{A\}$
- The surface of circle  $M \cap$  the surface of circle  $N =$  the surface of circle  $N$

If  $MN < r_1 - r_2$

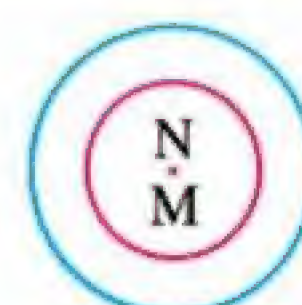


Then the two circles are :  
One inside the other  
(the circle  $N$  is inside the circle  $M$ )

Notice in the two cases that :

- The circle  $M \cap$  the circle  $N = \emptyset$
- The surface of circle  $M \cap$  the surface of circle  $N =$  the surface of circle  $N$

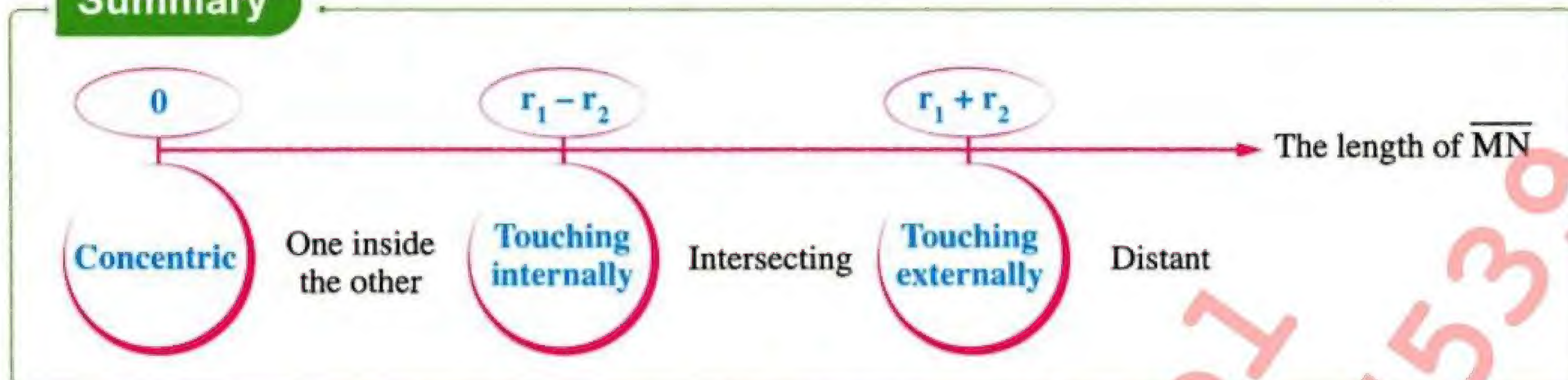
If  $MN = \text{zero}$



Then the two circles are :  
Concentric



## Summary



## Remarks

From the previous summary , we notice that :

- ① If M and N are two distant circles , then :  $MN \in ] r_1 + r_2 , \infty [$
- ② If M and N are two intersecting circles , then :  $MN \in ] r_1 - r_2 , r_1 + r_2 [$
- ③ If M and N (one of them is inside the other) , then :  $MN \in ] 0 , r_1 - r_2 [$

## Corollary 1

The line of centres of two touching circles passes through the point of tangency and is perpendicular to the common tangent at this point.

In the two opposite figures :

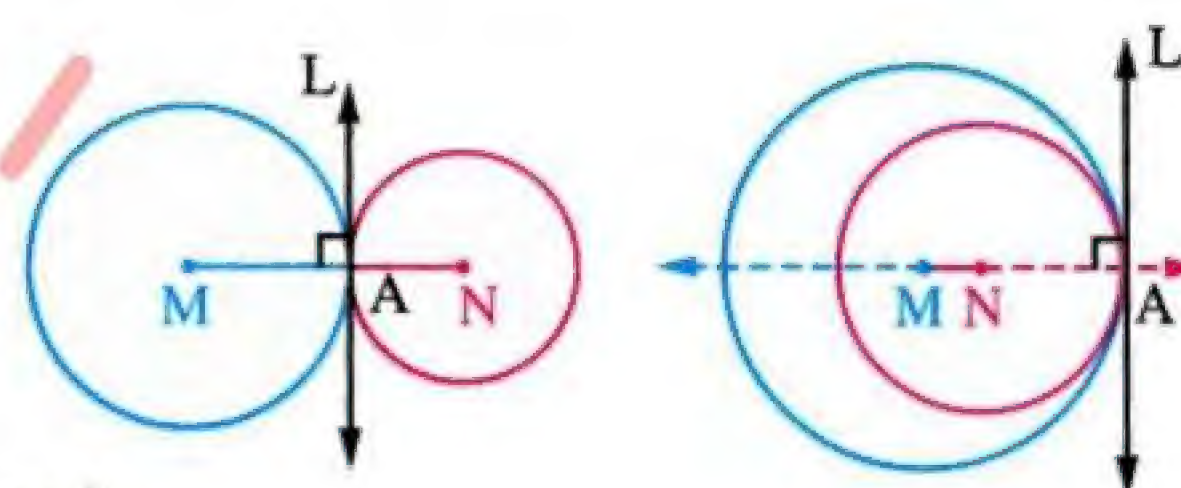
If the two circles

M and N are touching

at A (the point of tangency) ,

the straight line L is a common tangent to them at A

, then  $A \in \overleftrightarrow{MN}$  and  $\overleftrightarrow{MN} \perp$  the straight line L



## Corollary 2

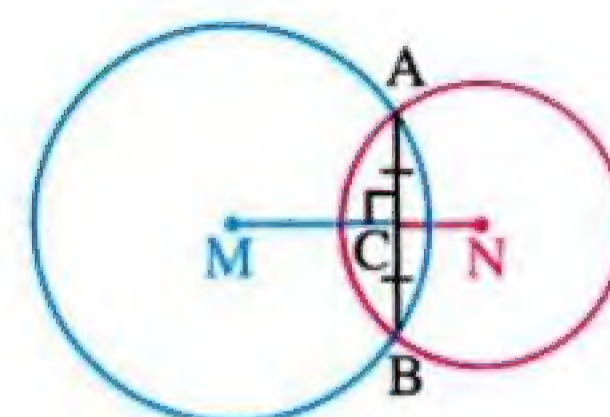
The line of centres of two intersecting circles is perpendicular to the common chord and bisects it.

In the opposite figure :

If M and N are two circles intersecting at A and B ,

then  $\overleftrightarrow{MN} \perp \overline{AB}$  ,  $\overleftrightarrow{MN}$  bisects  $\overline{AB}$  i.e.  $AC = BC$

This mean that  $\overleftrightarrow{MN}$  is the axis of symmetry of  $\overline{AB}$





# Exercises

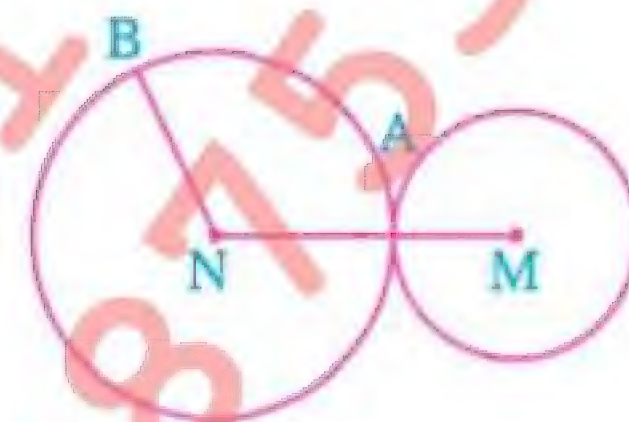
## [ A ] Essay problems : -

In the opposite figure :

- 1 M and N are two circles touching at A ,  
the distance between their centres  $MN = 12$  cm.  
If  $NB = 7$  cm.

**Find :** The length of  $\overline{MA}$

(Kafir El-Sheikh 06) « 5 cm. »



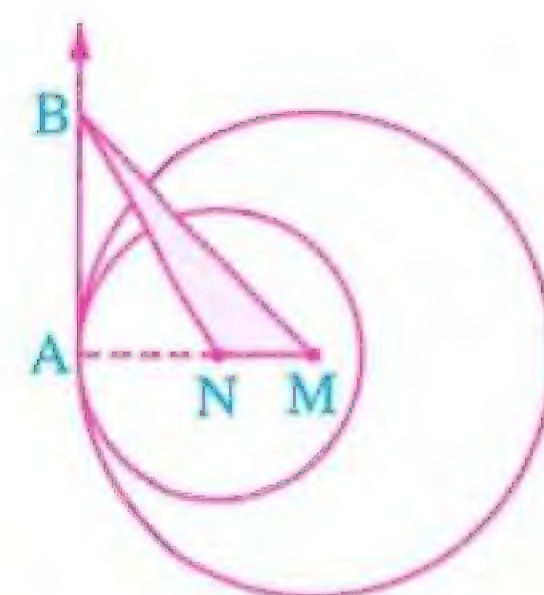
In the opposite figure :

- 2 M and N are two circles with radii lengths of 10 cm. and 6 cm.  
respectively and they are touching internally at A ,  
 $\overrightarrow{AB}$  is a common tangent for both.

If the area of  $\triangle BMN = 24$  cm<sup>2</sup>.

**Find :** The length of  $\overline{AB}$

(El-Kalyoubia 18 , Luxor 16 , Port Said 14) « 12 cm. »

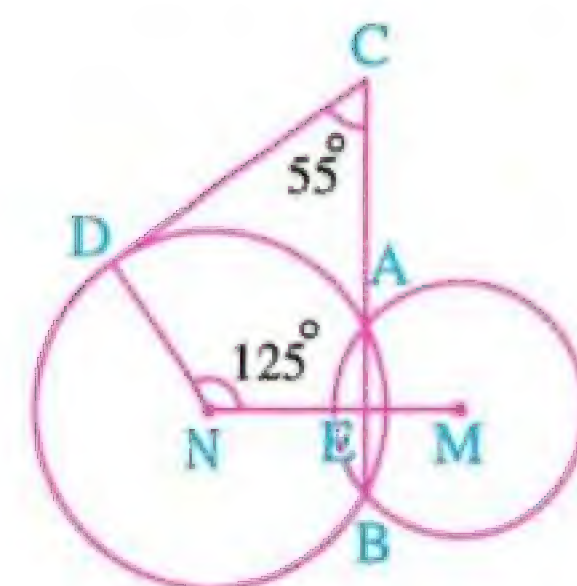


In the opposite figure :

- 3 M and N are two intersecting circles at A and B ,  
 $C \in \overrightarrow{BA}$  ,  $D \in$  the circle N ,  
 $m(\angle MND) = 125^\circ$  and  $m(\angle BCD) = 55^\circ$

**Prove that :**  $\overrightarrow{CD}$  is a tangent to circle N at D

(Red Sea 19 , Kafir El-Sheikh 17 , Souhag 15)



**In the opposite figure :**

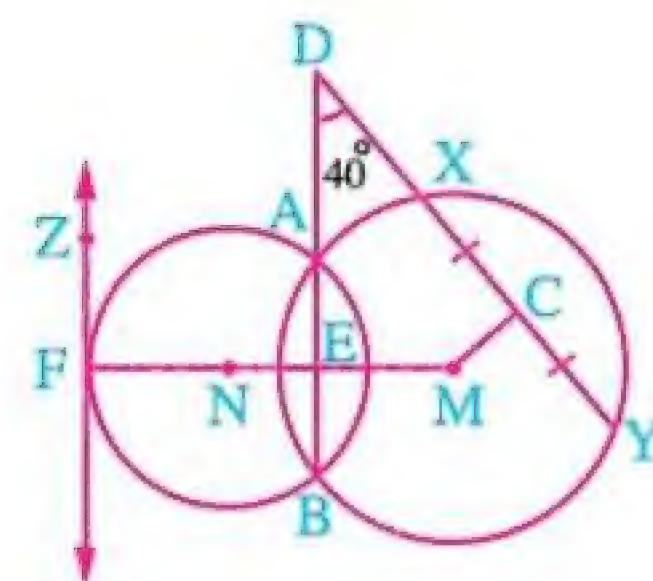
- 4 M and N are two intersecting circles at A and B ,  
C is the midpoint of  $\overline{XY}$  ,  $m(\angle D) = 40^\circ$  ,  
 $\overrightarrow{FZ}$  is a tangent to the circle N at F where  $\overrightarrow{MN} \cap \overrightarrow{FZ} = \{F\}$

**1 Find :**  $m(\angle CME)$

« 140° »

**2 Prove that :**  $\overrightarrow{FZ} \parallel \overline{AB}$

(El-Fayoum 11)



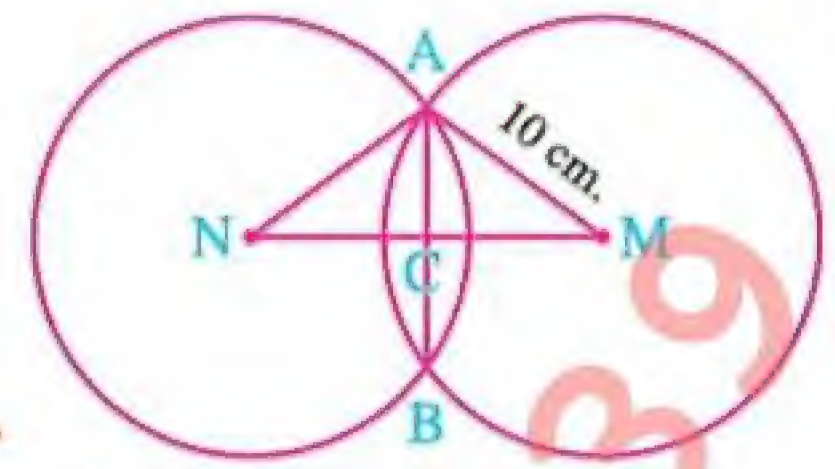


**In the opposite figure :**

Two congruent circles M and N are intersecting at A and B

If  $MA = 10 \text{ cm.}$  ,  $AB = 12 \text{ cm.}$

**Find by proof :** The length of  $\overline{MN}$  (El-Menia 17) « 16 cm. »



**M** and N are two intersecting circles at A and B ,  $MA = 12 \text{ cm.}$  ,  $NA = 9 \text{ cm.}$  and

$MN = 15 \text{ cm.}$

**Find :** The length of  $\overline{AB}$  (Port Said 11) « 14.4 cm. »

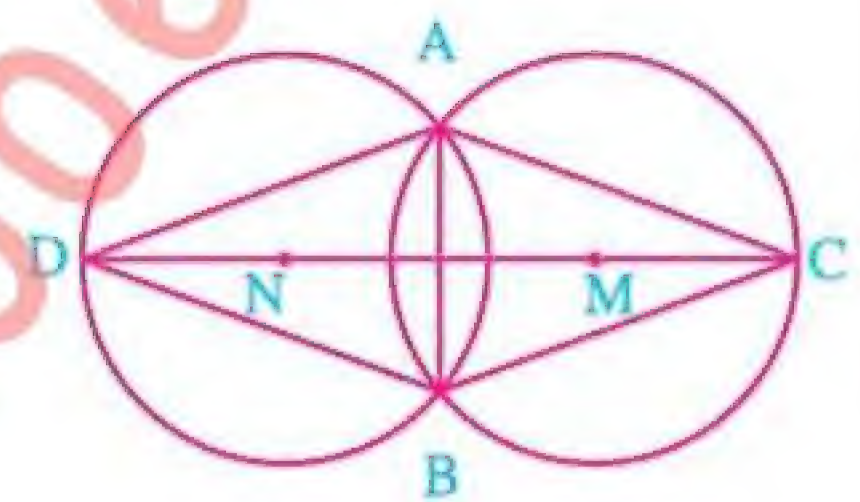
**In the opposite figure :**

M and N are two intersecting circles at A and B

where C is a point on the circle M ,

D is a point on the circle N ,  $C \in \overrightarrow{MN}$  ,  $D \in \overrightarrow{MN}$

**Prove that :**  $m(\angle CAD) = m(\angle CBD)$  (El-Sharkia 15)



If M (3 , 5) and N (− 3 , − 7) are the two centres of two circles whose radii lengths are  $4\sqrt{5}$  length units and  $2\sqrt{5}$  length units respectively , A (− 1 , − 3)


**Prove that :** The two circles are touching at A showing the kind of tangency. (Helwan 09)



**[ B ] Choose the correct : -**

1	<p>M and N are two circles touching internally , their radii lengths are 3 cm. and 5 cm., then <math>MN = \dots\dots\dots</math> cm. <span style="float: right;">(Beni Suef 17 , El-Gharbia 15)</span></p> <p>(a) 8                                      (b) 6                                      (c) 4                                      (d) 2</p>
2	<p>M and N are two circles touching externally , if their radii lengths are 4 cm. and 2 cm. , then <math>MN = \dots\dots\dots</math> cm. <span style="float: right;">(Cairo 15)</span></p> <p>(a) zero                                      (b) 2                                      (c) 6                                      (d) 7</p>
3	<p>M and N are two circles of radii lengths are 9 cm. and 4 cm. respectively , <math>MN = 5</math> cm. , then the two circles are <math>\dots\dots\dots</math> <span style="float: right;">(El-Dakahlia 17 , El-Gharbia 14)</span></p> <p>(a) touching externally.                                      (b) touching internally. (c) intersecting.                                      (d) distant.</p>
4	<p>M and N are two circles , their radii lengths are 8 cm. and 3 cm. , if <math>MN = 11</math> cm. , then the two circles M and N are <math>\dots\dots\dots</math> <span style="float: right;">(El-Menia 13)</span></p> <p>(a) distant.                                      (b) concentric. (c) intersecting.                                      (d) touching externally.</p>
5	<p>M and N are two circles , their radii lengths are 4 cm. and 3 cm. If <math>MN = 9</math> cm. , then the two circles are <math>\dots\dots\dots</math> <span style="float: right;">(Port Said 09)</span></p> <p>(a) distant.                                      (b) intersecting. (c) touching.                                      (d) one is inside the other.</p>
6	<p>If the radii lengths of the two circles M and N are 6 cm. , 3 cm. , if <math>MN = 2</math> cm. , then the two circles M , N are <math>\dots\dots\dots</math> <span style="float: right;">(El-Dakahlia 18)</span></p> <p>(a) intersecting.                                      (b) one is inside the other. (c) touching externally.                                      (d) distant.</p>



7	<p>If the radius length of the circle <math>M = 3</math> cm. and the radius length of the circle <math>N = 5</math> cm. , <math>MN = 6</math> cm. , then the two circles <math>M</math> and <math>N</math> are .....</p> <p>(a) distant. (b) one is inside the other. (c) intersecting. (d) touching externally.</p> <p style="text-align: right;">(El-Gharbia 08)</p>
8	<p> <math>M</math> and <math>N</math> are two intersecting circles their radii lengths are 3 cm. and 5 cm. respectively , then <math>MN \in</math> .....</p> <p>(a) <math>]0, 2[</math> (b) <math>]2, 8[</math> (c) <math>]8, \infty[</math> (d) <math>]2, \infty[</math></p> <p style="text-align: right;">(Alexandria 16 , Cairo 16 , Suez 11)</p>
9	<p>Two circles <math>M</math> and <math>N</math> with radii lengths 8 cm. and 5 cm. respectively , are touching when <math>MN \in</math> .....</p> <p>(a) <math>]13, 3[</math> (b) <math>]3, 13[</math> (c) <math>\mathbb{R} - [3, 13]</math> (d) <math>\{13, 3\}</math></p> <p style="text-align: right;">(El-Dakahlia 16)</p>
10	<p><math>M</math> and <math>N</math> are two intersecting circles at <math>A</math> and <math>B</math> , then the axis of symmetry of <math>\overline{AB}</math> is .....</p> <p>(a) <math>\overline{MN}</math> (b) <math>\overrightarrow{NM}</math> (c) <math>\overleftrightarrow{MN}</math> (d) <math>\overrightarrow{MN}</math></p> <p style="text-align: right;">(El-Monofia 04)</p>
11	<p>If the radius length of the circle <math>M =</math> the radius length of the circle <math>N = MN</math> , then the two circles are .....</p> <p>(a) one is inside the other. (b) touching externally. (c) distant. (d) intersecting.</p> <p style="text-align: right;">(Alexandria 05)</p>
12	<p>If the two circles <math>M</math> and <math>N</math> are touching internally , the radius length of one of them is 3 cm. and <math>MN = 8</math> cm. , then the radius length of the other circle = ..... cm.</p> <p>(a) 12 (b) 11 (c) 6 (d) 5</p> <p style="text-align: right;">(Giza 17)</p>
13	<p><math>M</math> and <math>N</math> are two touching circles where <math>MN = 6</math> cm. , the radius length of the greater circle is 10 cm. , then the radius length of the smaller circle = ..... cm.</p> <p>(a) 16 (b) 12 (c) 8 (d) 4</p> <p style="text-align: right;">(El-Sharkia 05)</p>



14	<p>M , N and L are three circles touching externally two-by-two, their radii lengths are 5 cm., 6 cm. and 4 cm., then the perimeter of the triangle MNL = ..... cm. (El-Monofia 11)</p> <p>(a) 15                      (b) 30                      (c) 4                      (d) 60</p>
15	<p>If the two circles M and N are touching externally , the radius length of the circle M is 4 cm. , if <math>MN = 7</math> cm. , then the circumference of the circle N is ..... cm. (El-Monofia 16)</p> <p>(a) <math>4\pi</math>                      (b) <math>6\pi</math>                      (c) <math>7\pi</math>                      (d) <math>\pi</math></p>
16	<p>A circle M of radius length 4 cm. touches a circle N internally , <math>MN = 7</math> cm. , then the circumference of the circle M : the circumference of the circle N = ..... (El-Dakahlia 09)</p> <p>(a) 4 : 7                      (b) 3 : 4                      (c) 4 : 3                      (d) 4 : 11</p>



## Solutions

A	Essay Problems
1	$\therefore MN = MA + NA$ $\therefore NA = NB = 7 \text{ cm. (lengths of two radii)}$ $\therefore 12 = MA + 7 \quad \therefore MA = 5 \text{ cm. (The req.)}$
2	$\therefore$ The two circles are touching internally at A $\therefore MN = 10 - 6 = 4 \text{ cm.}, \overline{MN} \perp \overline{AB}$ $\therefore$ The area of $\triangle BMN = \frac{1}{2} \times MN \times AB$ $\therefore 24 = \frac{1}{2} \times 4 \times AB \quad \therefore AB = 12 \text{ cm. (The req.)}$
3	$\therefore \overline{MN}$ is the line of centres, $\overline{AB}$ is the common chord $\therefore \overline{AB} \perp \overline{MN} \quad \therefore m(\angle AEN) = 90^\circ$ $\therefore$ The sum of the measures of the interior angles of the quadrilateral CDNE = $360^\circ$ $\therefore m(\angle CDN) = 360^\circ - (55^\circ + 125^\circ + 90^\circ) = 90^\circ$ $\therefore \overline{ND} \perp \overline{CD}$ $\therefore \overline{CD}$ is a tangent to the circle N at D (Q.E.D.)
4	$\therefore \overline{NM}$ is the line of centres, $\overline{AB}$ is the common chord $\therefore \overline{MN} \perp \overline{AB} \quad \therefore m(\angle AEM) = 90^\circ$ $\therefore C$ is the midpoint of $\overline{XY}$ $\therefore \overline{MC} \perp \overline{XY} \quad \therefore m(\angle MCX) = 90^\circ$ In the quadrilateral DCME : $m(\angle CME) = 360^\circ - (90^\circ + 90^\circ + 40^\circ) = 140^\circ$ (First req.) $\therefore \overline{FZ}$ is a tangent to the circle N at F $\therefore \overline{NF} \perp \overline{FZ} \quad \therefore m(\angle NFZ) = 90^\circ$ $\therefore m(\angle MEA) = m(\angle NFZ)$ and they are corresponding angles $\therefore \overline{FZ} \parallel \overline{AB}$ (Second req.)
5	$\therefore \overline{MN}$ is the line of centres, $\overline{AB}$ is the common chord of the two circles $\therefore \overline{MN} \perp \overline{AB} \quad \therefore C$ is the midpoint of $\overline{AB}$ $\therefore AC = \frac{1}{2} \times 12 = 6 \text{ cm.}$ $\therefore MC = \sqrt{(AM)^2 - (AC)^2} = \sqrt{100 - 36} = 8 \text{ cm.}$ In $\triangle AMN$ :

	$\therefore AM = AN = r \quad \therefore \overline{AC} \perp \overline{MN}$ $\therefore C$ is the midpoint of $\overline{MN}$ $\therefore MN = 2 MC = 2 \times 8 = 16 \text{ cm. (The req.)}$
6	$\therefore \overline{MN}$ is the line of centres, $\overline{AB}$ is the common chord of the two circles $\therefore \overline{MN} \perp \overline{AB}, AC = CB$ In $\triangle AMN : (AN)^2 = 81$ $\therefore (AM)^2 = 144, (MN)^2 = 225$ $\therefore (MN)^2 = (AM)^2 + (AN)^2$ $\therefore \triangle AMN$ is right-angled at A, $\therefore \overline{AC} \perp \overline{MN}$ $\therefore AC = \frac{AM \times AN}{MN} = \frac{12 \times 9}{15} = 7.2 \text{ cm.}$ $\therefore AB = 2 AC = 14.4 \text{ cm. (The req.)}$
7	$\therefore \overline{MN}$ is the line of centres, $\overline{AB}$ is the common chord $\therefore \overline{MN}$ is the axis of symmetry of $\overline{AB}$ $\therefore CA = CB$ $\therefore$ In $\triangle ABC : m(\angle CAB) = m(\angle CBA) \quad (1)$ $\therefore DA = DB$ $\therefore$ In $\triangle ABD : m(\angle DAB) = m(\angle DBA) \quad (2)$ By adding (1), (2) : $\therefore m(\angle CAD) = m(\angle CBD) \quad (Q.E.D.)$
8	$\therefore MA = \sqrt{(3+1)^2 + (5+3)^2} = \sqrt{16+64}$ $= 4\sqrt{5} \text{ length unit}$ $\therefore A \in \text{the circle M}$ $\therefore NA = \sqrt{(-3+1)^2 + (-7+3)^2} = \sqrt{4+16}$ $= 2\sqrt{5} \text{ length units}$ $\therefore A \in \text{the circle N}$ $\therefore MN = \sqrt{(3+3)^2 + (5+7)^2} = \sqrt{36+144}$ $= \sqrt{180} = 6\sqrt{5} \text{ length units}$ $\therefore MN = MA + NA$ $\therefore$ The two circles are touching externally. (Q.E.D.)



B	Choose
1	D
2	C
3	B
4	D
5	A
6	B
7	C
8	B
9	D
10	C
11	D
12	B
13	D
14	B
15	B

16

D



الصف الثالث الإعدادي

الترم الثاني - هندسة

Prep. [ 3 ]

Second Term

Geometry

4

Sheet

Mr. Mahmoud Esamiel

01110882717

01006487539



# Prep [ 3 ] - Second Term - Geometry - Unit [ 4 ] - The Circle

## Lesson [ 4 ] : Identifying The Circle

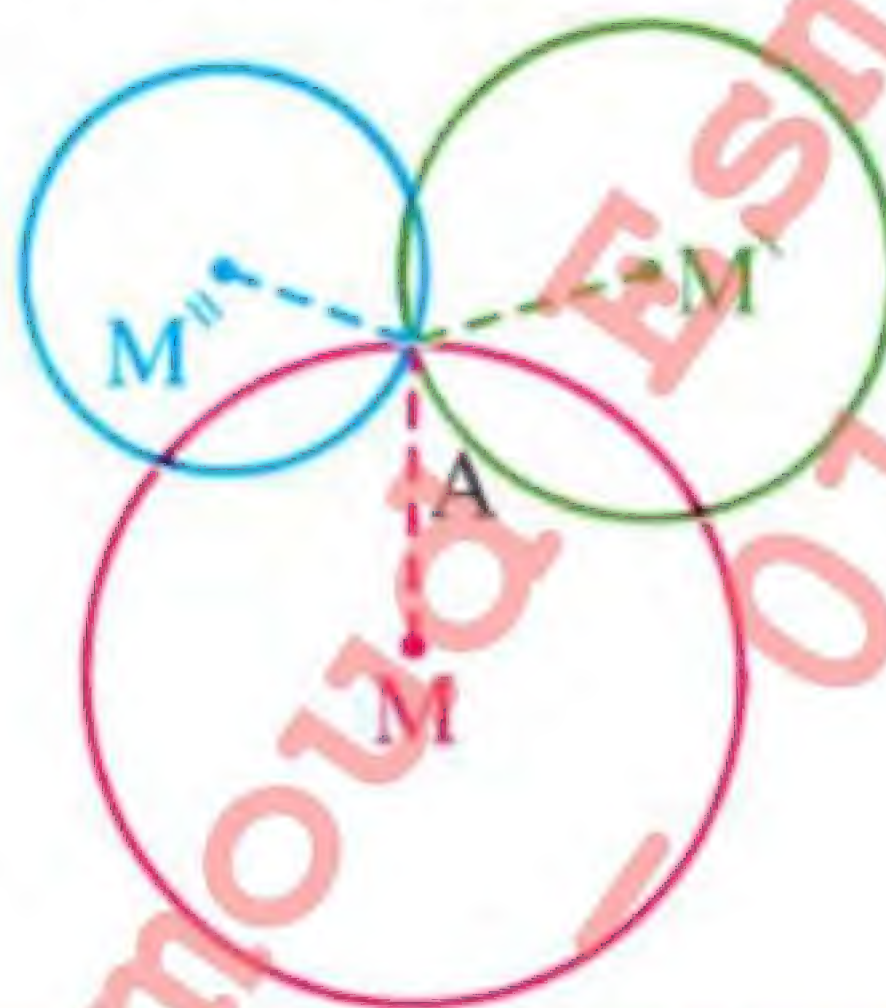
The circle is identified if we know :

- 1 its centre.
- 2 its radius length.

In the following, we will study the possibility of identifying (drawing) the circle under certain conditions.

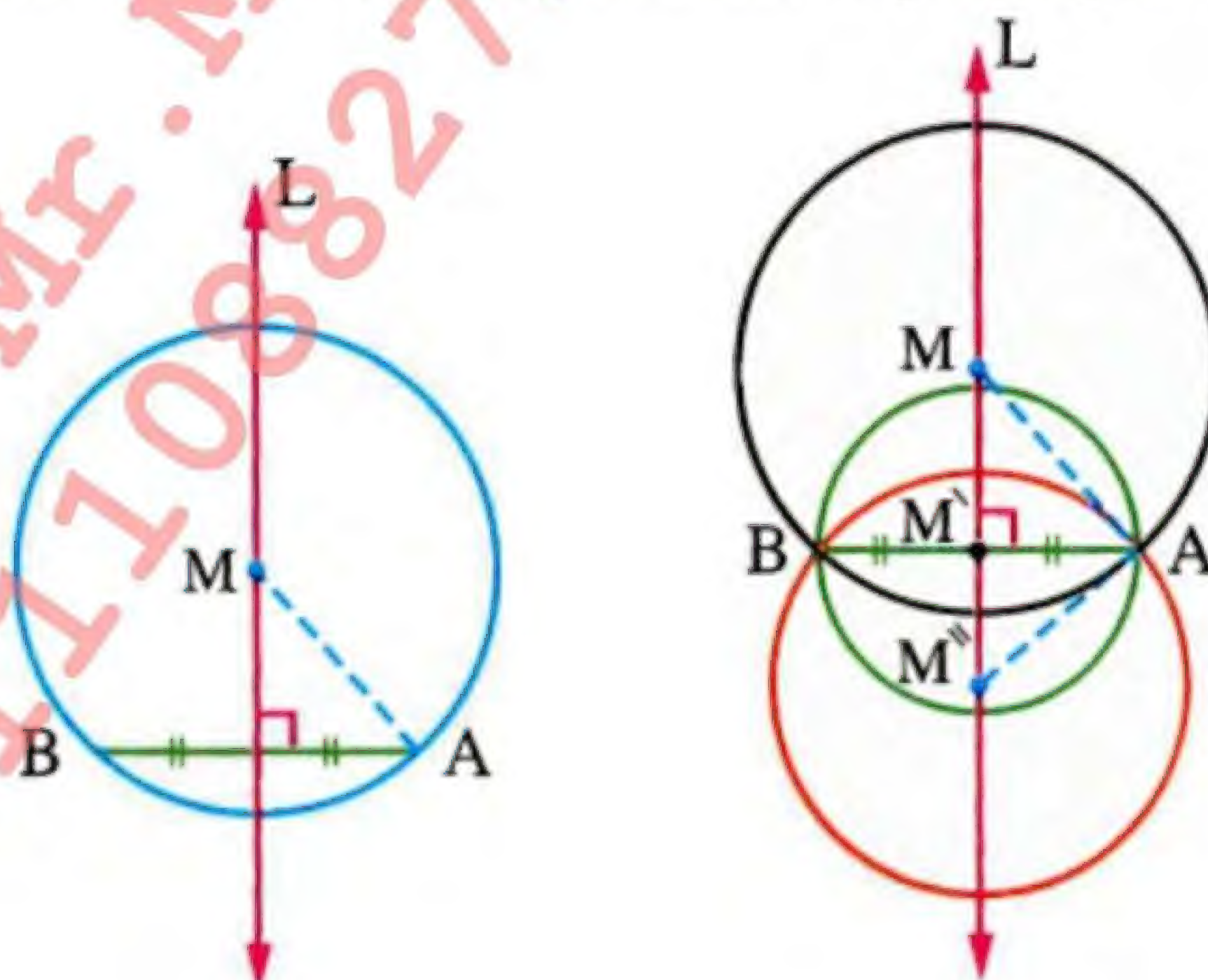
### First Drawing a circle passing through a given point

We can draw an infinite number of circles passing through a given point.



### Second Drawing a circle passing through two given points

There is an infinite number of circles that can be drawn to pass through the two points A and B and all their centres lie on the axis of symmetry of  $\overline{AB}$





## Remarks

• If  $\overline{AB}$  is a line segment and the required is drawing a circle passing through the two points A and B , then :

① If  $r > \frac{1}{2} AB$  , then we can draw two circles (as shown in the previous example).

② If  $r = \frac{1}{2} AB$  , then we can draw one and only one circle (it is the smallest circle) passing through the two points A and B , hence  $\overline{AB}$  is a diameter of it and its centre is the midpoint of  $\overline{AB}$

③ If  $r < \frac{1}{2} AB$  , then it is impossible to draw any circle.

• Any two circles do not intersect at more than two points.

”

## Third Drawing a circle passing through three given points

If A , B and C are three points in the plane and the required is drawing a circle passing through the three points A , B and C :

Then we must distinguish between two cases :

1 If the points A , B and C are collinear as in figure (1) , then the two straight lines  $L_1$  and  $L_2$  are parallel not intersecting.

In this case , it is impossible to draw a circle passing through the three points A , B and C

i.e.

It is impossible to draw a circle passing through three collinear points.

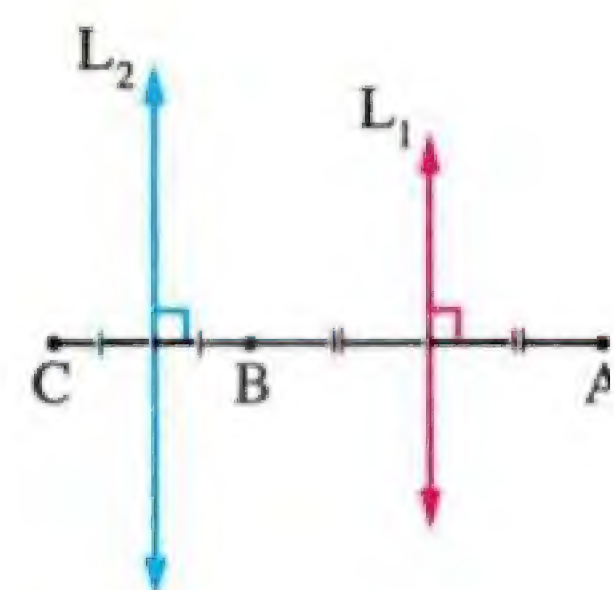


Fig (1)

2 If the points A , B and C are not collinear as in figure (2) , then  $L_1$  and  $L_2$  intersect at one point as M , then M is the centre of the required circle which passes through the three points A , B and C , then the radius length of this circle =  $MA = MB = MC$

i.e.

For any three non-collinear points , there is a unique circle can be drawn to pass through them.

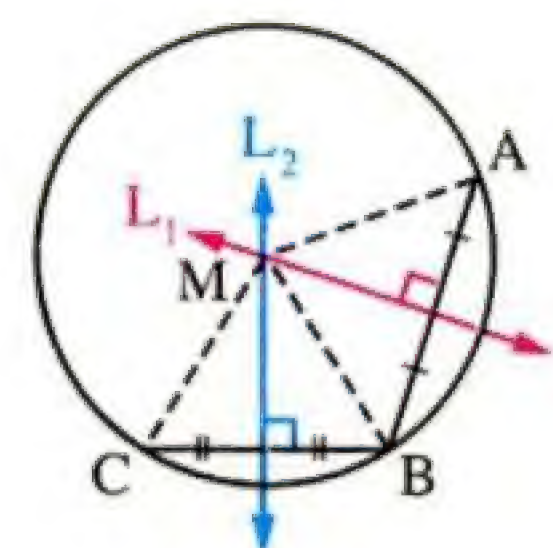


Fig (2)



**Notice that :**

There is a unique circle passing through three points as A , B and C which are not collinear and the centre of this circle is the point of intersection of any two axes of symmetry of the axes of the line segments  $\overline{AB}$  ,  $\overline{BC}$  and  $\overline{AC}$

**Corollary 1**

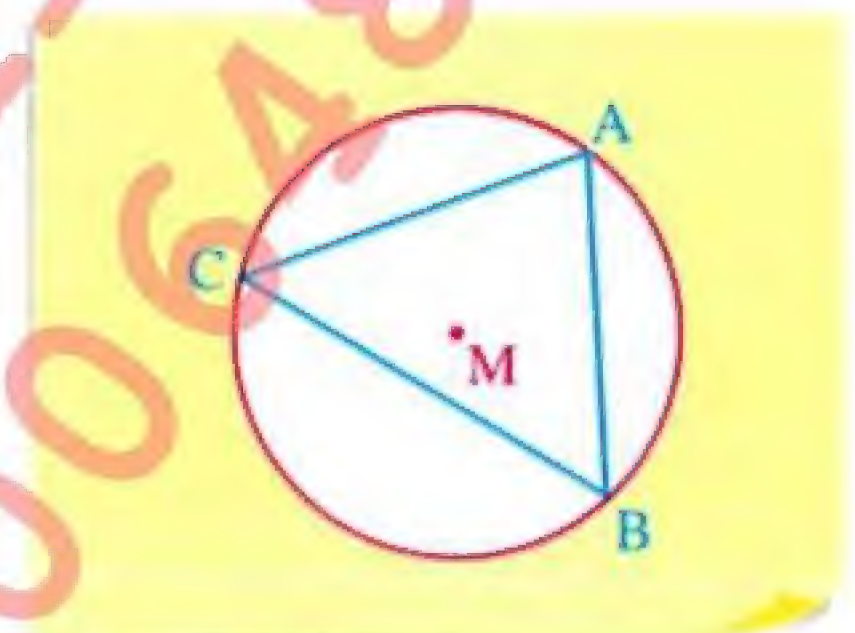
The circle which passes through the vertices of a triangle is called the **circumcircle** of this triangle.

- The triangle whose vertices lie on a circle is called the inscribed triangle of this circle.

In the opposite figure :

**M** is the circumcircle of  $\triangle ABC$

or  $\triangle ABC$  is the inscribed triangle of the circle **M**

**Corollary 2**

The perpendicular bisectors of the sides of a triangle intersect at a point which is the centre of the circumcircle of the triangle.

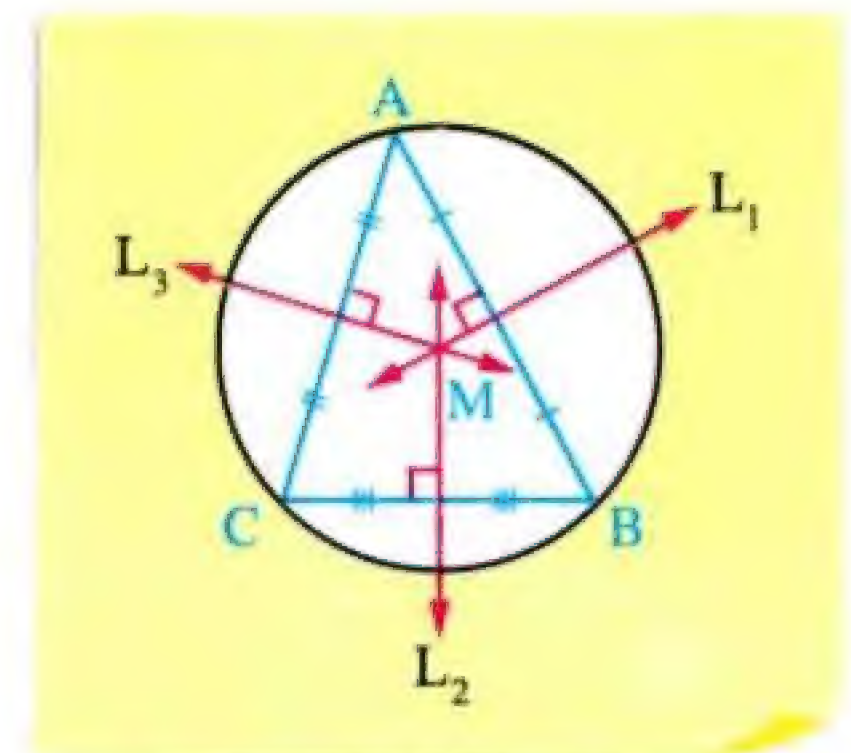
In the opposite figure :

If the straight lines  $L_1$  ,  $L_2$  and  $L_3$

are the axes of  $\overline{AB}$  ,  $\overline{BC}$  and  $\overline{CA}$  respectively

and  $L_1 \cap L_2 \cap L_3 = \{M\}$  ,

then the point M is the centre of the circumcircle of  $\triangle ABC$

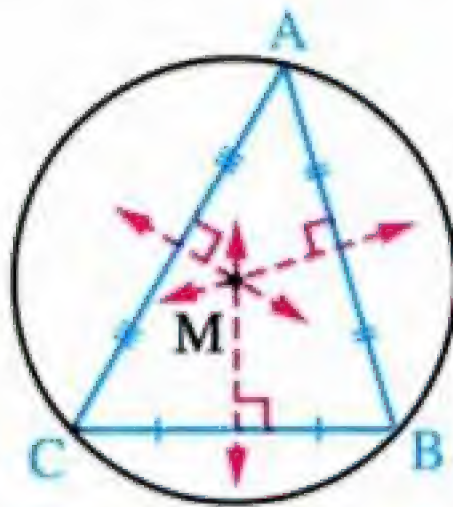




### Remark

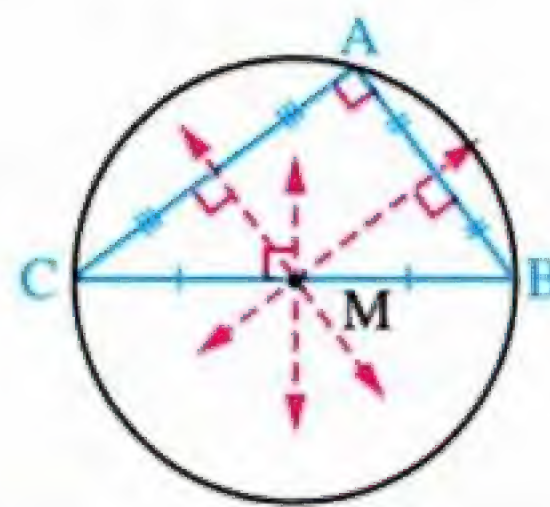
The position of the centre of the circumcircle of the triangle as M differs according to the type of the triangle as shown in the following table :

The acute-angled triangle



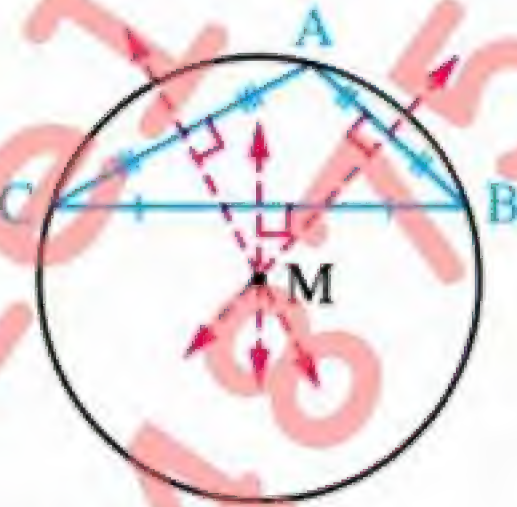
M is inside the triangle

The right-angled triangle



M is the midpoint of the hypotenuse

The obtuse-angled triangle

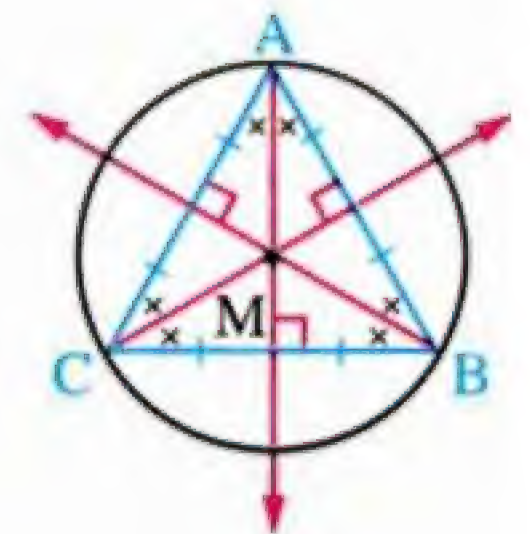


M is outside the triangle

### • A special case :

The centre of the circumcircle of the equilateral triangle is :

- The point of intersection of its sides axes.
- The point of intersection of its altitudes.
- The point of intersection of its medians.
- The point of intersection of the bisectors of its interior angles.




### Remark

We can draw a circle passing through the vertices of (the rectangle , the square or the isosceles trapezium) while we cannot draw a circle passing through the vertices of (the parallelogram , the rhombus or the trapezium which is not isosceles).



# Exercises

## [ A ] Essay problems : -

1	<p>If <math>A \in L</math> , draw the circle <math>M</math> passing through <math>A</math> and its radius length = 3 cm. if :</p> <p>1 <math>M \in</math> the straight line <math>L</math> , how many circles can be drawn ?</p> <p>2 <math>M \notin</math> the straight line <math>L</math> , how many circles can be drawn ? <span style="float: right;">(Assiut 11)</span></p>
2	<p><math>A</math> and <math>B</math> are two points where <math>AB = 6</math> cm. Draw a circle of radius length 5 cm. and passes through the two points <math>A</math> and <math>B</math></p> <p>Find :</p> <p>1 The number of circles can be drawn.</p> <p>2 The distance of the centre of the circle from <math>\overline{AB}</math> by proof. <span style="float: right;">(Damietta 17) « 4 cm. »</span></p>
3	<p><math>\overline{AB}</math> is a line segment of length 6 cm. Draw the circle that passes through the two points <math>A</math> and <math>B</math> and its radius length is the smallest length. <span style="float: right;">(Luxor 05)</span></p>
4	<p>Using the geometric tools and draw <math>\overline{AB}</math> with length 6 cm. , then draw <math>\overrightarrow{AC}</math> where <math>m(\angle CAB) = 60^\circ</math> , draw the circle that passes through the points <math>A</math> , <math>B</math> and its centre lies on <math>\overrightarrow{AC}</math> and calculate the length of its radius (Don't remove the arcs). <span style="float: right;">(El-Dakahlia 17) « 6 cm. »</span></p>
5	<p>Draw a circle with radius length of 3 cm. and touches to the straight line <math>L</math></p> <p>What is the number of possible solutions ? <span style="float: right;">(Giza 06)</span></p>
6	<p> Draw the right-angled triangle <math>ABC</math> at <math>B</math> where <math>AB = 4</math> cm. and <math>BC = 3</math> cm. , then draw the circumcircle of this triangle. Where does the centre of the circle lie with respect to the sides of this triangle ? <span style="float: right;">(Damietta 18)</span></p>
7	<p>Using geometrical instruments, draw the isosceles triangle <math>ABC</math> in which <math>m(\angle ABC) = 120^\circ</math> , <math>BC = 4</math> cm. Determine the centre of the circumcircle of it and find its radius length. <span style="float: right;">(El-Dakahlia 11) « 4 cm. »</span></p>
8	<p>Draw <math>\triangle ABC</math> in which : <math>AB = 5</math> cm. , <math>BC = 4</math> cm. , and <math>CA = 3</math> cm. What is the type of the triangle with respect to the measures of its angles ? then draw a circle whose centre is the point <math>A</math> and touches <math>\overleftrightarrow{BC}</math> , another circle whose centre is <math>B</math> and touches <math>\overleftrightarrow{AC}</math> and a third circle whose centre is <math>C</math> and touches <math>\overleftrightarrow{AB}</math> <span style="float: right;">(Beni Suef 06)</span></p>



- 9 If A (2 , 0) and B (− 2 , 3) , draw a circle M of radius length 4 length units and passes through the two points A and B  
How many solutions are there for this problem ?

(North Sinai 09)

Mr. Mahmoud Esmail  
01110882717 - 01006487539



**[ B ] Choose the correct : -**

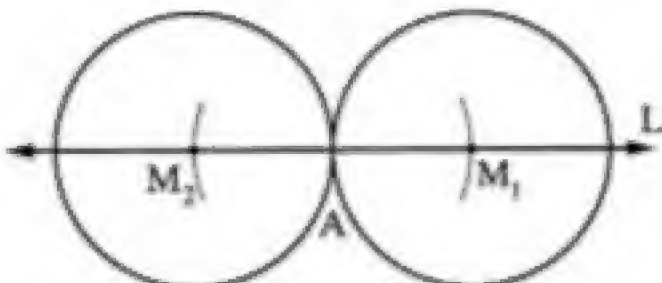
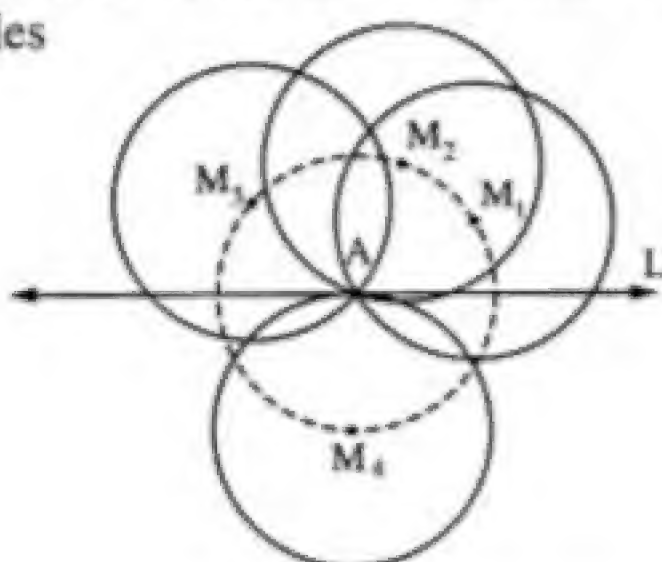
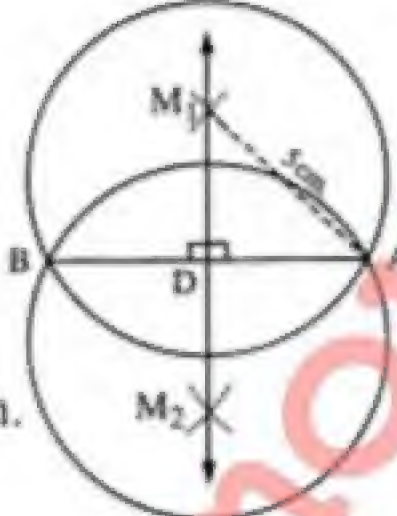

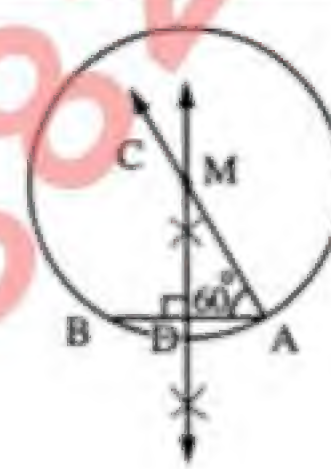
1	It is possible to draw ..... passing through a given point. <span style="float: right;">(New Valley 05)</span> (a) one circle (b) two circles (c) three circles (d) an infinite number of circles
2	The number of circles which passes through two given points is ..... <span style="float: right;">(Giza 12)</span> (a) 1 (b) 2 (c) 3 (d) an infinite number.
3	The number of circles passing through three collinear points is ..... <span style="float: right;">(Sohag 18 , Giza 16 , Ismailia 15)</span> (a) zero (b) one (c) three (d) an infinite number.
4	The number of circles passing through three non-collinear points is ..... <span style="float: right;">(El-Menia 17)</span> (a) 1 (b) zero (c) 2 (d) 3
5	We can identify the circle if we are given ..... <span style="float: right;">(El-Sharkia 08)</span> (a) three collinear points. (b) two points. (c) three non-collinear points. (d) one point.
6	The centres of the circles passing through the two points A and B lie on ..... <span style="float: right;">(El-Dakahlia 17)</span> (a) the axis of symmetry of $\overline{AB}$ (b) $\overline{AB}$ (c) the perpendicular to $\overline{AB}$ (d) the midpoint of $\overline{AB}$
7	The centre of the circumcircle of a triangle is the point of intersection of ..... <span style="float: right;">(El-Fayoum 19 , Kafr El-Sheikh 17 , Qena 17)</span> (a) the bisectors of its interior angles. (b) the bisectors of its exterior angles. (c) its altitudes. (d) the symmetry axes of its sides.


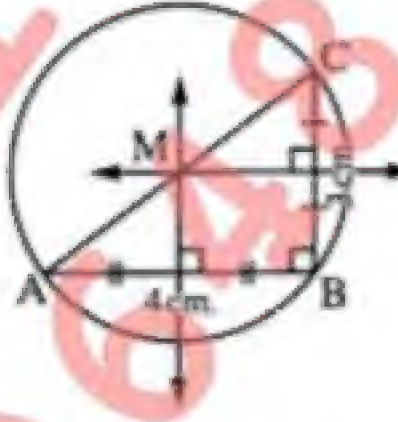
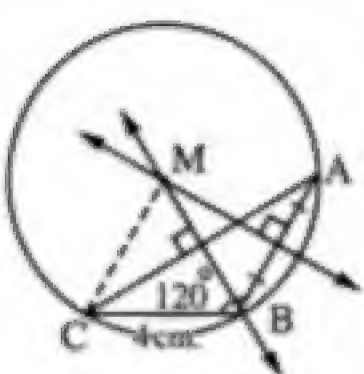
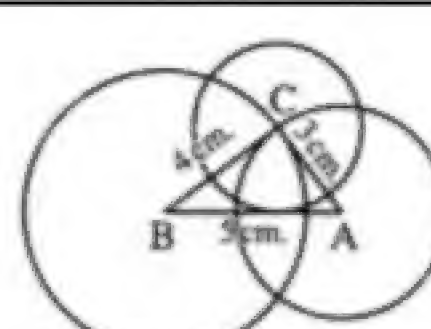
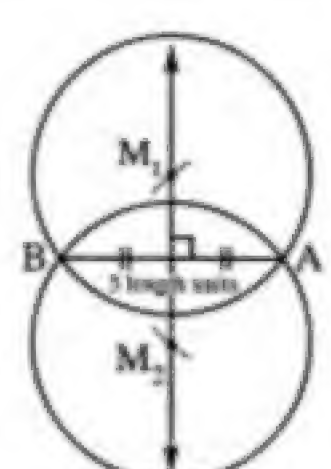


8	<p>If <math>\Delta ABC</math> is right-angled at B , then the centre of its circumcircle is ..... <i>(Ismailia 03)</i></p> <p>(a) the midpoint of <math>\overline{AB}</math> (b) the midpoint of <math>\overline{AC}</math>  (c) the midpoint of <math>\overline{BC}</math> (d) outside the triangle.</p>	
9	<p>It is (impossible) to draw a circle passing through the vertices of ..... <i>(Beni Suef 17 , El-Dakahlia 13 , El-Sharkia 12)</i></p> <p>(a) a rectangle. (b) a triangle. (c) a square. (d) a rhombus.</p>	
10	<p>It is possible to draw a circle passing through the vertices of ..... <i>(El-Sharkia 19 , Sohag 18 , Giza 17 , Beni Suef 16)</i></p> <p>(a) a rhombus. (b) a rectangle. (c) a trapezium. (d) a parallelogram.</p>	
11	<p>If <math>\overline{AB}</math> is a line segment of length 4 cm. , then the radius length of the smallest circle which passes through the two points A and B = ..... cm. <i>(El-Monofia 16)</i></p> <p>(a) 2 (b) 3 (c) 4 (d) 5</p>	
12	<p>If <math>AB = 6</math> cm. , then the area of the smallest circle which passes through the two points A and B = ..... <math>\text{cm}^2</math> <i>(El-Sharkia 15)</i></p> <p>(a) <math>3\pi</math> (b) <math>6\pi</math> (c) <math>8\pi</math> (d) <math>9\pi</math></p>	



## Solutions

A	Essay Problems
1	<p>1 When <math>M \in L</math> we can draw two circles</p>  <p>2 When <math>M \notin L</math> we can draw an infinite number of circles</p> 
2	<p>1 We can draw two circles</p> <p>2 In <math>\triangle AM_1D</math> :</p> <p><math>\therefore \overline{M_1D} \perp \overline{AB}</math></p> <p><math>\therefore D</math> is the midpoint of <math>\overline{AB}</math></p> <p><math>\therefore AD = \frac{1}{2} AB = \frac{1}{2} \times 6 = 3 \text{ cm.}</math></p> <p><math>\therefore m(\angle ADM_1) = 90^\circ</math></p> <p><math>\therefore M_1D = \sqrt{(AM_1)^2 - (AD)^2}</math></p> <p><math>= \sqrt{25 - 9} = 4 \text{ cm.}</math> (The req.)</p> 
3	<p><math>\therefore</math> The radius length is the smallest</p> <p><math>\therefore r = 3 \text{ cm.}</math></p> 
4	<p>In <math>\triangle ADM</math> : <math>\therefore \overline{AD} \perp \overline{MD}</math></p> <p><math>\therefore D</math> is the midpoint of <math>\overline{AB}</math></p> <p><math>\therefore AD = \frac{1}{2} AB = \frac{1}{2} \times 6 = 3 \text{ cm.}</math></p> <p><math>\therefore m(\angle ADM) = 90^\circ</math></p> <p><math>\therefore m(\angle A) = 60^\circ</math></p> <p><math>\therefore m(\angle AMD) = 180^\circ - (90^\circ + 60^\circ) = 30^\circ</math></p> <p><math>\therefore AM = 2 AD = 2 \times 3 = 6 \text{ cm.}</math> (The req.)</p> 
5	<p>There are an infinite number of circles whose centres lie on a straight line parallel to the straight line L at a distance 3 cm. from it.</p>

	
6	 <p>The centre of the circle lies at the midpoint of the hypotenuse <math>\overline{AC}</math></p>
7	<p>In <math>\triangle ABC</math> :</p> <p><math>\therefore AB = BC</math></p> <p><math>\therefore \triangle ABC</math> is an isosceles triangle</p> <p><math>\therefore \overline{BM} \perp \overline{AC}</math></p> <p><math>\therefore \overline{BM}</math> bisects <math>\angle ABC</math> <math>\therefore m(\angle MBC) = 60^\circ</math></p> <p><math>\therefore MB = MC = r</math></p> <p><math>\therefore \triangle MBC</math> is an equilateral triangle</p> <p><math>\therefore MB = MC = BC = r = 4 \text{ cm.}</math> (The req.)</p> 
8	 <p>The type of this triangle according to the measures of its angle is right-angled triangle at C</p>
9	<p><math>AB = \sqrt{(2+2)^2 + (0-3)^2}</math></p> <p><math>= \sqrt{16+9} = \sqrt{25}</math></p> <p><math>= 5 \text{ length units}</math></p> <p><math>\therefore</math> There are two solutions</p> 
B	Choose
1	D



2	D
3	A
4	A
5	C
6	A
7	D
8	B
9	D
10	B
11	A
12	D



الصف الثالث الإعدادي

الترم الثاني - هندسة

Prep. [ 3 ]

Second Term

5

Geometry

Sheet

Mr. Mahmoud Esamiel

01110882717

01006487539



## Prep [ 3 ] - Second Term - Geometry - Unit [ 4 ] - The Circle

## Lesson [ 5 ] : The Relation Between The Chords Of A Circle With Its Center

Fig. (1)

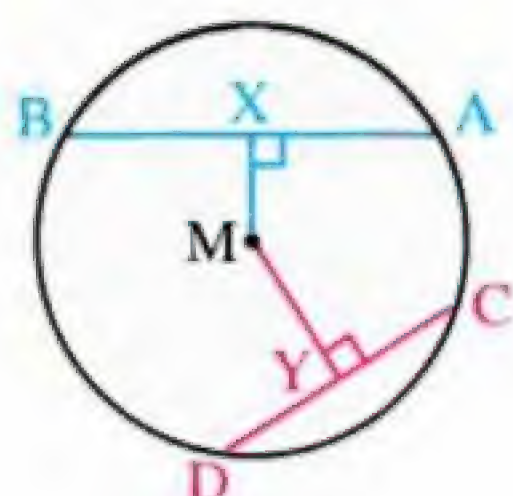


Fig. (2)

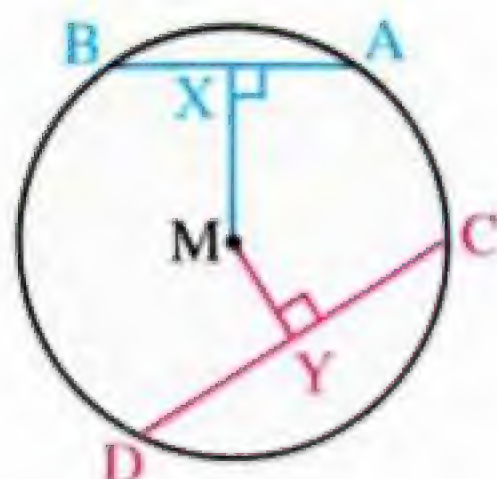
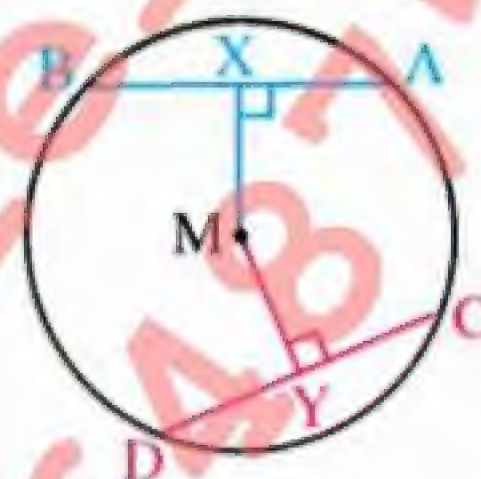


Fig. (3)



Using the ruler , you can check by yourself the truth of the following information :

$$AB > CD \\ , MX < MY$$

$$AB < CD \\ , MX > MY$$

$$AB = CD \\ , MX = MY$$

The relation between the chords of a circle and its centre :

## Theorem

If chords of a circle are equal in length, then they are equidistant from the centre.

**Given**  $AB = CD$  ,  $\overline{MX} \perp \overline{AB}$  and  $\overline{MY} \perp \overline{CD}$

**R.T.P.**  $MX = MY$

**Construction** Draw  $\overline{MA}$  and  $\overline{MC}$

**Proof**  $\therefore \overline{MX} \perp \overline{AB}$

$\therefore X$  is the midpoint of  $\overline{AB}$

$$\therefore AX = \frac{1}{2} AB$$

$\therefore \overline{MY} \perp \overline{CD} \quad \therefore Y$  is the midpoint of  $\overline{CD}$

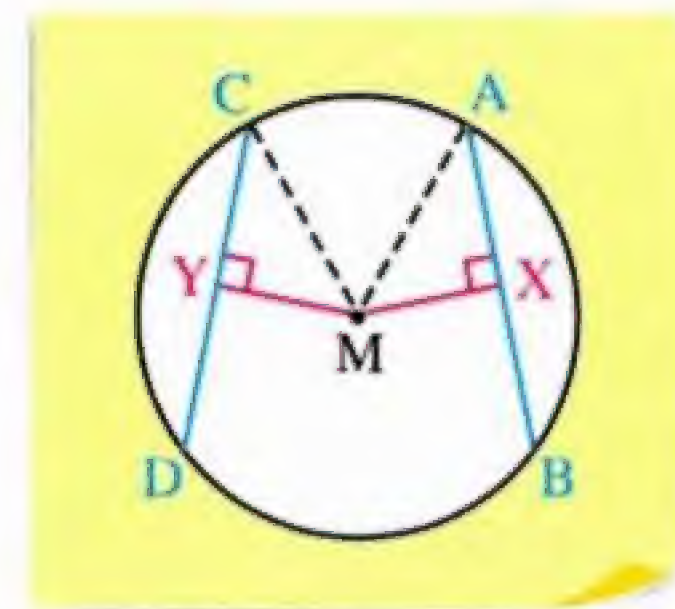
$$\therefore CY = \frac{1}{2} CD$$

$\therefore AB = CD$  (given)  $\therefore AX = CY$

$\therefore \triangle AXM$  and  $\triangle CYM$  , both have  $\begin{cases} AX = CY \text{ (by proof)} \\ MA = MC = r \\ m(\angle AXM) = m(\angle CYM) = 90^\circ \end{cases}$

$\therefore \triangle AXM \cong \triangle CYM$  , then we get :  $MX = MY$

(Q.E.D.)





**Corollary**

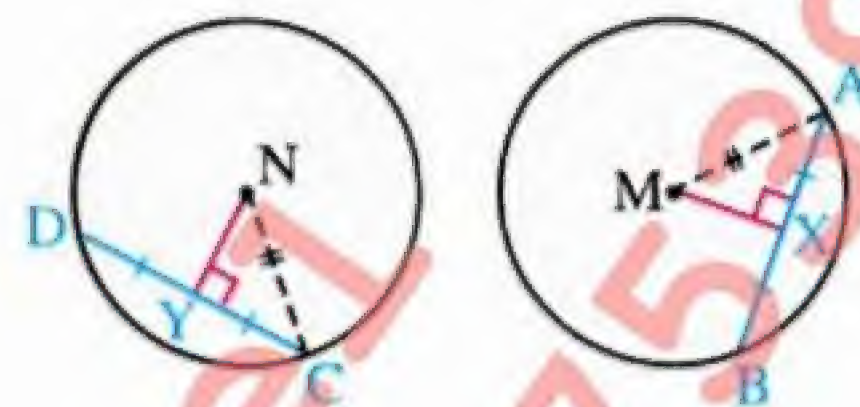
**In congruent circles, chords which are equal in length are equidistant from the centres.**

**In the opposite figure :**

If M and N are two congruent circles ,

$AB = CD$  ,  $\overline{MX} \perp \overline{AB}$  and  $\overline{NY} \perp \overline{CD}$  ,

then  $MX = NY$

**Converse of the theorem**

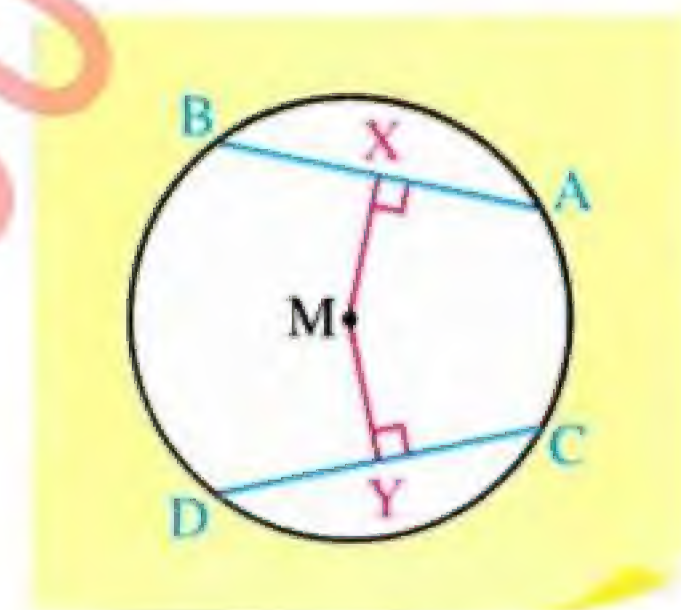
**In the same circle (or in congruent circles) ,**

**chords which are equidistant from the centre (s) are equal in length.**

**i.e. In the opposite figure :**

If  $\overline{AB}$  and  $\overline{CD}$  are two chords of the circle M ,

$\overline{MX} \perp \overline{AB}$  ,  $\overline{MY} \perp \overline{CD}$  and  $MX = MY$  , then  $AB = CD$

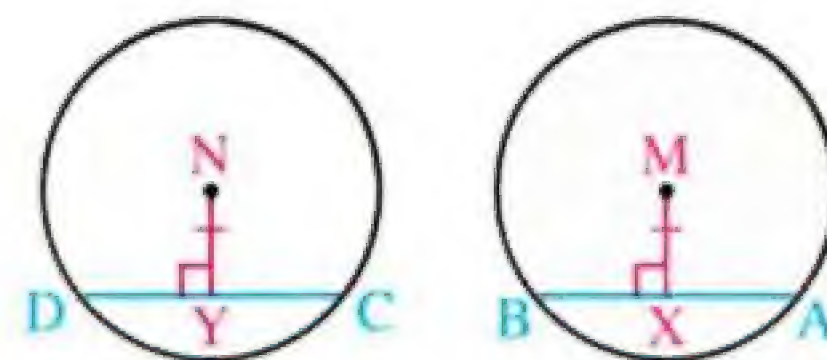


**Also in the opposite figure :**

If M and N are two congruent circles ,  $\overline{AB}$  is a chord of circle M and  $\overline{CD}$  is a chord of circle N

,  $\overline{MX} \perp \overline{AB}$  ,  $\overline{NY} \perp \overline{CD}$  and

$MX = NY$  , then  $AB = CD$





# Exercises

## [ A ] Essay problems : -

In the opposite figure :

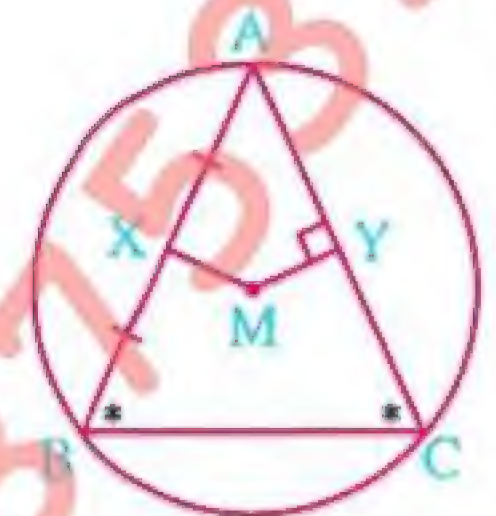
The triangle ABC is an inscribed triangle inside a circle M ,

$m(\angle B) = m(\angle C)$  ,

X is the midpoint of  $\overline{AB}$  ,  $\overline{MY} \perp \overline{AC}$

**Prove that :**  $MX = MY$

(Giza 19 , El-Beheira 19 , Matrouh 17 , Fayoum 15)



In the opposite figure :

M is a circle,  $m(\angle A) = 60^\circ$

, X is the midpoint of  $\overline{AC}$

, Y is the midpoint of  $\overline{BC}$

,  $FX = EY$

**Prove that :**  $\triangle ABC$  is an equilateral triangle

(El-Sharkia 18)



In the opposite figure :

$\overline{AB}$  and  $\overline{AC}$  are two chords equal in length in the circle M

, X is the midpoint of  $\overline{AB}$  ,

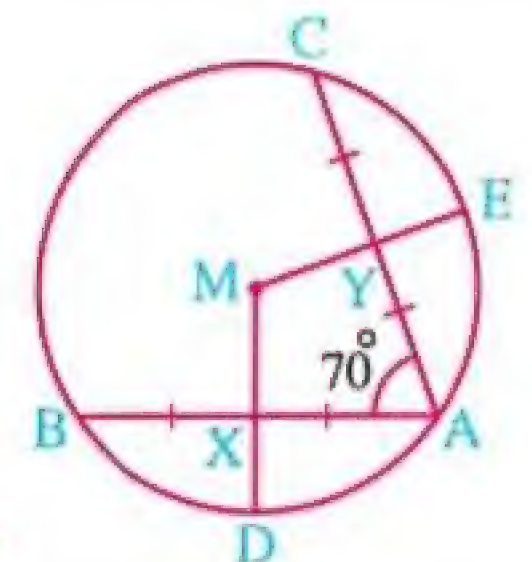
Y is the midpoint of  $\overline{AC}$  and  $m(\angle CAB) = 70^\circ$

**1 Calculate :**  $m(\angle DME)$

**2 Prove that :**  $XD = YE$

« 110° »

(New Valley 19 , Port said 18 , Matrouh 18 , Cairo 17)



In the opposite figure :

$\overline{AB}$  and  $\overline{AC}$  are two chords equal in length in the circle M

, X is the midpoint of  $\overline{AB}$  ,

$\overline{MX}$  intersects the circle at D ,  $\overline{MY} \perp \overline{AC}$

intersects it at Y and intersects the circle at E

**Prove that :** **1**  $XD = YE$

**2**  $m(\angle YXB) = m(\angle XYC)$

(Assiut 18 , El-Gharbia 13)



$\overline{AB}$  and  $\overline{AC}$  are two chords equal in length in the circle M , X and Y are the midpoints of  $\overline{AB}$  and  $\overline{AC}$  respectively ,  $m(\angle MXY) = 30^\circ$

**Prove that :** **1**  $\triangle MXY$  is an isosceles triangle.

**2**  $\triangle AXY$  is an equilateral triangle.

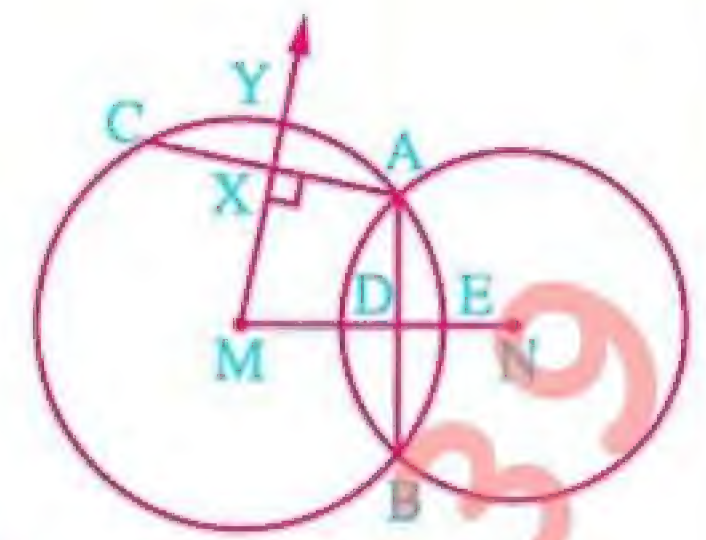
(New Valley 16)



**In the opposite figure :**

M and N are two circles intersecting at A and B  
 $\overrightarrow{MX} \perp \overline{AC}$  and intersects  $\overline{AC}$  at X and intersects  
the circle M at Y ,  $\overrightarrow{MN}$  to intersects  $\overline{AB}$  at D and  
intersects the circle M at E, if  $AC = AB$

**Prove that :**  $XY = DE$



(El-Kalyoubia 18)

$\overline{AB}$  and  $\overline{AC}$  are two chords in the circle M ,  $\overrightarrow{MX} \perp \overline{AB}$  , Y is the midpoint of  $\overline{AC}$  ,  
 $m(\angle ABC) = 75^\circ$  ,  $MX = MY$

**1 Find :**  $m(\angle BAC)$

«  $30^\circ$  »

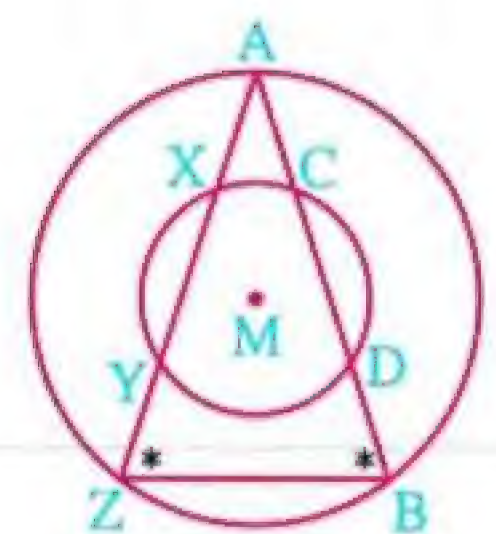
**2 Prove that :** The perimeter of  $\triangle AXY = \frac{1}{2}$  the perimeter of  $\triangle ABC$

(Kaf El-Sheikh 18 , Alexandria 16)

**In the opposite figure :**

Two concentric circles at M ,  $\overline{AB}$  is a chord  
in the greater circle and cuts the smaller circle  
at C and D ,  $\overline{AZ}$  is a chord in the greater circle  
and cuts the smaller circle at X and Y If  $m(\angle ABZ) = m(\angle AZB)$

**Prove that :**  $CD = XY$



(El-Kalyoubia 17 , Souhag 13)

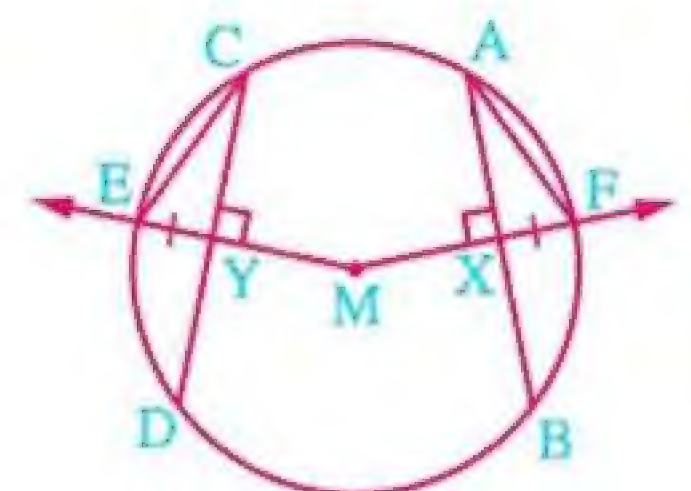
**In the opposite figure :**

$\overline{AB}$  and  $\overline{CD}$  are two chords of the circle M ,  
 $\overrightarrow{MX} \perp \overline{AB}$  and intersects the circle at F ,  
 $\overrightarrow{MY} \perp \overline{CD}$  and intersects the circle at E ,  
 $FX = EY$

**Prove that :**

**1**  $AB = CD$

**2**  $AF = CE$



(El-Gharbia 16 , Kaf El-Sheikh 11)



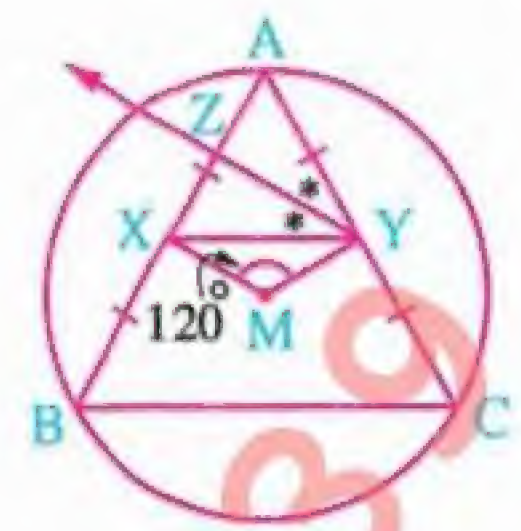
**In the opposite figure :**

$\overline{AB}$  and  $\overline{AC}$  are two chords of the circle M , equal in length

X and Y are their midpoints respectively.

If  $m(\angle XMY) = 120^\circ$  ,  $\overrightarrow{YZ}$  bisects  $\angle AYX$

**Prove that :**  $\overrightarrow{YZ} \parallel \overrightarrow{MX}$



(Cairo 08)

**In the opposite figure :**

The circle M  $\cap$  the circle N = { A , B } ,  $\overline{AB} \cap \overline{MN} = \{ C \}$  ,

$D \in \overline{MN}$  ,  $\overline{MX} \perp \overline{AD}$  and  $\overline{MY} \perp \overline{BD}$

**Prove that :**  $MX = MY$



(El-Kalyoubia 19 , El-Sharkia 11)

**In the opposite figure :**

The concentric circles of radii 4 cm. , 2 cm.

$\Delta ABC$  is drawn such that its vertices lie

on the greater circle and its sides touch

the smaller circle at X , Y , Z

**Prove that :**

$\Delta ABC$  is an equilateral triangle and find its area.



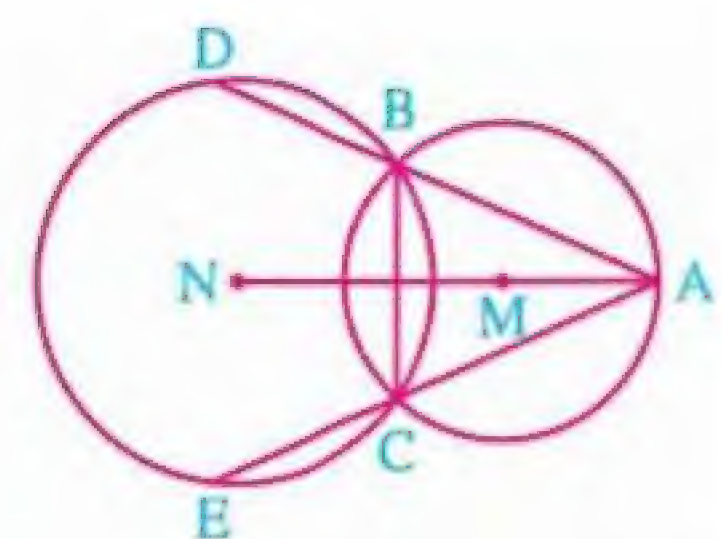
(El-Fayoum 19) «  $12\sqrt{3} \text{ cm}^2$  »

**In the opposite figure :**

M , N are two intersecting circles at B , C

,  $A \in \overline{MN}$

**Prove that :**  $BD = CE$



(El-Dakahlia 17)

**In the opposite figure :**

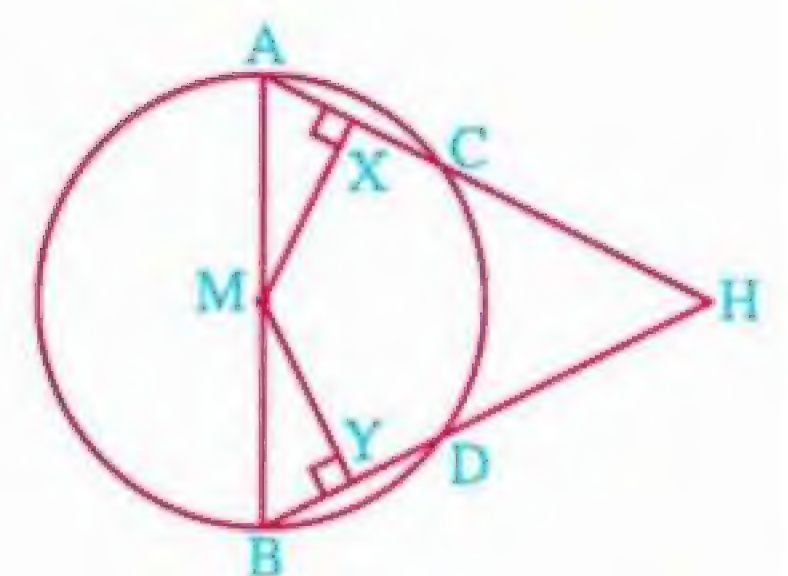
$\overline{AB}$  is a diameter of the circle M ,  $\overline{AC}$  and  $\overline{BD}$  are two chords in it ,

$MX = MY$  ,  $\overline{MX} \perp \overline{AC}$  ,  $\overline{MY} \perp \overline{DB}$

**Prove that :**

1  $\Delta HAB$  is isosceles triangle.

2  $HC = HD$



(Beni Suef 12)



**In the opposite figure :**

$\Delta ABC$  is inscribed in the circle  $M$  ,  
 $m(\angle BAC) = 60^\circ$  ,  $X$  is the midpoint of  $\overline{AB}$  ,  
 $Y$  is the midpoint of  $\overline{AC}$  and  $MX = MY$

**Prove that :**

- 1  $ABC$  is an equilateral triangle.      2  $\overline{AM} \perp \overline{BC}$



(Giza 05)

**In the opposite figure :**

$\overline{AB}$  and  $\overline{CD}$  are two chords of the circle  $M$  ,  
 equal in length ,  $X$  and  $Y$  are the two midpoints  
 of  $\overline{AB}$  and  $\overline{CD}$  respectively.  $\overline{XY}$  is drawn to cut  
 the circle at  $E$  and  $F$  ,  $\overline{ML}$  is drawn  $\perp \overline{XY}$

**Prove that :**  $XE = YF$

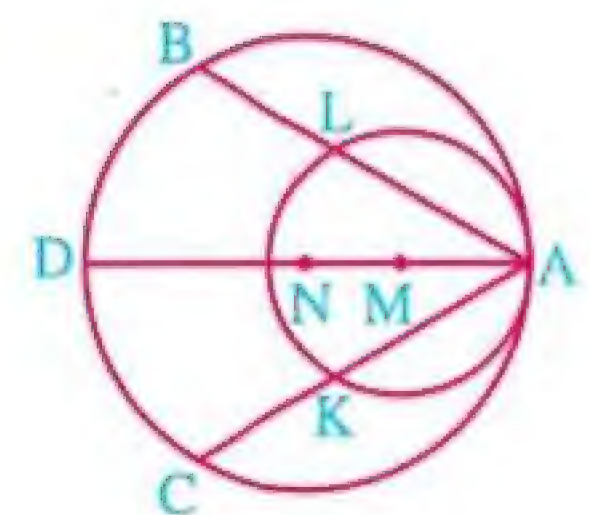


(Cairo 03)

**In the opposite figure :**

$M$  and  $N$  are two circles touching internally at  $A$  ,  
 $\overline{AB}$  and  $\overline{AC}$  are two chords drawn in  
 the greater circle  $N$  such that they are equal in length  
 to cut the smaller circle  $M$  at  $L$  and  $K$  respectively.

**Prove that :**  $AL = AK$

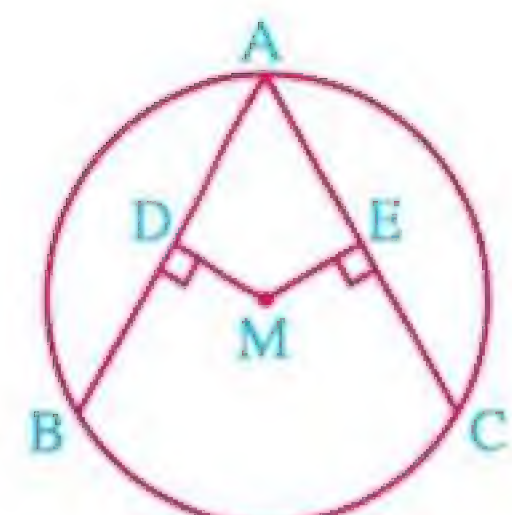


(Dakahlia 09)

**In the opposite figure :**

$M$  is a circle ,  $\overline{MD} \perp \overline{AB}$   
 $\overline{ME} \perp \overline{AC}$   
 $A(2, 2)$  ,  $D(1, 0)$  and  $E(3, 4)$

**Prove that :**  $ME = MD$

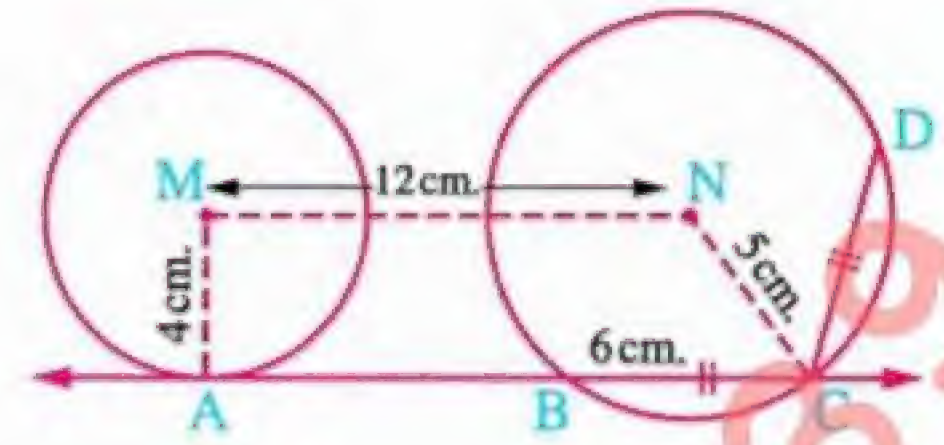


(Kaf El-Sheikh 13)



In the opposite figure :

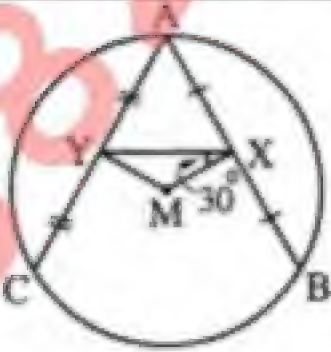
M and N are two circles of radii lengths 4 cm. and 5 cm. ,  $\overline{AC}$  touches the circle M at A and cuts the circle N at B and C , where  $BC = 6$  cm. and  $MN = 12$  cm.

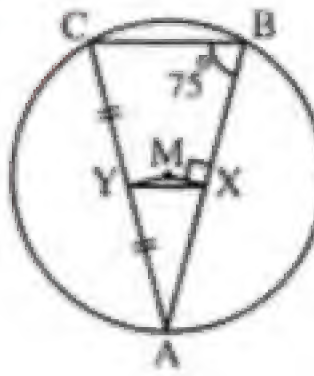



- 1 Prove that the quadrilateral MACN is a trapezium then calculate its area. « 54 cm<sup>2</sup> »
- 2 If  $CD = CB$  , find the distance between N and  $\overline{CD}$  (Sharkia 06) « 4 cm. »



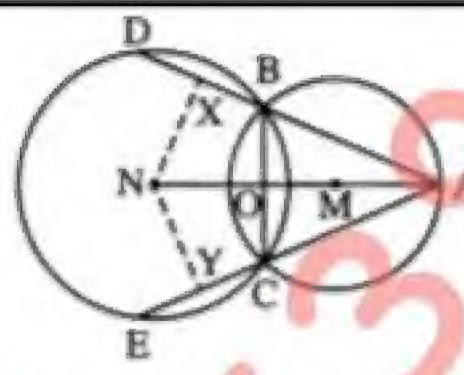
## Solutions

A	Essay Problems
1	<p>In <math>\triangle ABC</math> : <math>\because m(\angle B) = m(\angle C)</math>  <math>\therefore AB = AC</math>  <math>\because X</math> is the midpoint of <math>\overline{AB}</math> <math>\therefore \overline{MX} \perp \overline{AB}</math>  <math>\because \overline{MY} \perp \overline{AC}</math> , <math>AB = AC</math> <math>\therefore MX = MY</math> (Q.E.D.)</p>
2	<p><math>\because MF = ME</math> (lengths of two radii)  <math>\therefore FX = EY</math>          By subtracting : <math>\therefore MX = MY</math>  <math>\because X</math> is the midpoint of <math>\overline{AC}</math> <math>\therefore \overline{MX} \perp \overline{AC}</math>  <math>\because Y</math> is the midpoint of <math>\overline{BC}</math> <math>\therefore \overline{MY} \perp \overline{BC}</math>  <math>\therefore AC = BC</math> <math>\because m(\angle A) = 60^\circ</math>  <math>\therefore \triangle ABC</math> is an equilateral triangle. (Q.E.D.)</p>
3	<p><math>\because X</math> is the midpoint of <math>\overline{AB}</math> <math>\therefore \overline{MX} \perp \overline{AB}</math>  <math>\because Y</math> is the midpoint of <math>\overline{AC}</math> <math>\therefore \overline{MY} \perp \overline{AC}</math>  <math>\because</math> The sum of measures of the interior angles of the quadrilateral <math>AXMY = 360^\circ</math>  <math>\therefore m(\angle XMY) = 360^\circ - (70^\circ + 90^\circ + 90^\circ) = 110^\circ</math>          (First req.)  <math>\because AB = AC</math> <math>\therefore MX = MY</math>  <math>\because MD = ME</math> (lengths of two radii)          by subtracting <math>\therefore XD = YE</math> (Second req.)</p>
4	<p><math>\because X</math> is the midpoint of <math>\overline{AB}</math> <math>\therefore \overline{MX} \perp \overline{AB}</math>  <math>\because AB = AC</math> <math>\therefore MX = MY</math> ,  <math>\because MD = ME</math> (lengths of two radii) by subtracting  <math>\therefore XD = YE</math> (Q.E.D. 1)          In <math>\triangle XMY</math> : <math>\because MX = MY</math>  <math>\therefore m(\angle MXY) = m(\angle MYX)</math>  <math>\because m(\angle MXB) = m(\angle MYC) = 90^\circ</math>          by adding <math>\therefore m(\angle YXB) = m(\angle XYC)</math> (Q.E.D. 2)</p>
5	<p><math>\because X</math> is the midpoint of <math>\overline{AB}</math>  <math>\therefore \overline{MX} \perp \overline{AB}</math>  <math>\because Y</math> is the midpoint of <math>\overline{AC}</math>  <math>\therefore \overline{MY} \perp \overline{AC}</math> <math>\because AB = AC</math>  <math>\therefore MX = MY</math>  <math>\therefore \triangle MXY</math> is an isosceles triangle (Q.E.D. 1)  <math>\because m(\angle AXM) = 90^\circ</math> , <math>m(\angle MXY) = 30^\circ</math>  <math>\therefore m(\angle AXY) = 90^\circ - 30^\circ = 60^\circ</math>  <math>\because X</math> and <math>Y</math> are the midpoints of <math>\overline{AB}</math> and <math>\overline{AC}</math> , <math>AB = AC</math>  <math>\therefore AX = AY</math>  <math>\therefore \triangle AXY</math> is an equilateral triangle (Q.E.D. 2)</p> 

6	<p><math>\because \overline{AB}</math> is the common chord of the two circles M , N  <math>\therefore \overline{MN}</math> is the line of centres  <math>\therefore \overline{MN} \perp \overline{AB}</math> <math>\therefore \overline{MD} \perp \overline{AB}</math>  <math>\because \overline{MX} \perp \overline{AC}</math> , <math>AC = AB</math>  <math>\therefore MX = MD</math> (1)  <math>\because MY = ME</math> (lengths of two radii) (2)          Subtracting (1) from (2) :  <math>\therefore XY = DE</math> (Q.E.D.)</p>
7	<p><math>\because Y</math> is the midpoint of <math>\overline{AC}</math>  <math>\therefore \overline{MY} \perp \overline{AC}</math>  <math>\because \overline{MX} \perp \overline{AB}</math> , <math>MX = MY</math>  <math>\therefore AB = AC</math>  <math>\therefore m(\angle C) = 75^\circ</math>  <math>\therefore m(\angle A) = 180^\circ - (75^\circ + 75^\circ) = 30^\circ</math> (First req.)  <math>\therefore \overline{MX} \perp \overline{AB}</math> <math>\therefore X</math> is the midpoint of <math>\overline{AB}</math>  <math>\therefore</math> In <math>\triangle ABC</math> :  <math>XY = \frac{1}{2} BC</math> , <math>AX = \frac{1}{2} AB</math> , <math>AY = \frac{1}{2} AC</math>  <math>\therefore</math> The perimeter of <math>\triangle AXY</math>  <math>= \frac{1}{2}</math> The perimeter of <math>\triangle ABC</math> (Second req.)</p> 
8	<p><b>Constr. :</b>          Draw : <math>\overline{MF} \perp \overline{AB}</math> , <math>\overline{ME} \perp \overline{AZ}</math>  <b>Proof :</b> In the great circle :  <math>\because m(\angle ABZ) = m(\angle AZB)</math>  <math>\therefore AB = AZ</math>  <math>\because \overline{MF} \perp \overline{AB}</math> , <math>\overline{ME} \perp \overline{AZ}</math> <math>\therefore MF = ME</math>          In the small circle :  <math>\because \overline{MF} \perp \overline{CD}</math> , <math>\overline{ME} \perp \overline{XY}</math> , <math>MF = ME</math>  <math>\therefore CD = XY</math> (Q.E.D.)</p> 
9	<p><math>\because MF = ME</math> (lengths of two radii)  <math>\therefore XF = YE</math> <math>\therefore MX = MY</math>  <math>\because \overline{MX} \perp \overline{AB}</math> , <math>\overline{MY} \perp \overline{CD}</math> <math>\therefore AB = CD</math> (Q.E.D. 1)  <math>\because \overline{MX} \perp \overline{AB}</math>  <math>\therefore X</math> is the midpoint of <math>\overline{AB}</math> <math>\therefore AX = \frac{1}{2} AB</math>  <math>\because \overline{MY} \perp \overline{CD}</math>  <math>\therefore Y</math> is the midpoint of <math>\overline{CD}</math> <math>\therefore CY = \frac{1}{2} CD</math>  <math>\therefore AB = CD</math> <math>\therefore AX = CY</math></p>



	$\therefore \triangle AXF, \triangle CYE$ In them $\begin{cases} AX = CY \\ XF = YE \\ m(\angle AXF) = m(\angle CYE) = 90^\circ \end{cases}$ $\therefore \triangle AXF \cong \triangle CYE$ then we deduce that $AF = CE$ (Q.E.D. 2)
10	$\therefore Y$ is the midpoint of $\overline{AC} \therefore \overline{MY} \perp \overline{AC}$ (1) Similarly $\overline{MX} \perp \overline{AB}$ $\therefore AC = AB \therefore MY = MX$ and from $\triangle YMX : \therefore m(\angle M) = 120^\circ$ $\therefore m(\angle MYX) = m(\angle YXM) = \frac{180^\circ - 120^\circ}{2} = 30^\circ$ (2) from (1) and (2) : $\therefore m(\angle AYX) = 90^\circ - 30^\circ = 60^\circ$ $\therefore \overline{YZ}$ bisects $\angle AYX$ $\therefore m(\angle ZYX) = \frac{60^\circ}{2} = 30^\circ$ $\therefore m(\angle ZYX) = m(\angle YXM)$ but they are alternate angles $\therefore \overline{YZ} \parallel \overline{MX}$ (Q.E.D.)
11	$\therefore$ The circle $M \cap$ the circle $N = \{A, B\}$ $\therefore \overline{MN}$ is the axis of symmetry of $\overline{AB}$ $\therefore$ In $\triangle ABD : \overline{DC}$ is the axis of symmetry of $\overline{AB}$ $\therefore AD = BD$ $\therefore \overline{MX} \perp \overline{AD}, \overline{MY} \perp \overline{BD} \therefore MX = MY$ (Q.E.D.)
12	<b>Constr. :</b> Draw $\overline{MX}, \overline{MY}, \overline{MZ}, \overline{MA}$ <b>Proof :</b> $\therefore \overline{AB}$ is a tangent to the smaller circle $M$ $\therefore \overline{MX} \perp \overline{AB}$ similarly : $\overline{MY} \perp \overline{BC}, \overline{MZ} \perp \overline{AC}$ $\therefore MX = MY = MZ = r$ in the smaller circle $\therefore AB = BC = AC$ $\therefore \triangle ABC$ is an equilateral triangle (First req.) $\therefore m(\angle B) = 60^\circ$ $\therefore$ the greater circle $M$ is the circumcircle of $\triangle ABC$ $\therefore M$ is the point of intersection of the altitudes of $\triangle ABC$ $\therefore \overline{AY}$ is an altitude in $\triangle ABC$ $\therefore$ In $\triangle ABY$ which is right at $Y : \sin B = \frac{AY}{AB}$ $\therefore AY = AM + MY = 4 + 2 = 6$ cm. $\therefore \sin 60^\circ = \frac{6}{AB} \therefore \frac{\sqrt{3}}{2} = \frac{6}{AB}$ $\therefore AB = \frac{2 \times 6}{\sqrt{3}} = 4\sqrt{3}$ cm. $\therefore BC = AB = 4\sqrt{3}$ cm. $\therefore$ The area of $\triangle ABC = \frac{1}{2} \times BC \times AY$ $= \frac{1}{2} \times 4\sqrt{3} \times 6 = 12\sqrt{3}$ cm <sup>2</sup> (Second req.)

	<b>Constr. :</b> Draw $\overline{NX} \perp \overline{BD}$ $\overline{NY} \perp \overline{EC}$ 
13	<b>Proof :</b> $\therefore \overline{MN}$ is the line of centres $\therefore \overline{BC}$ is the common chord of the two circles $\therefore \overline{MN} \perp \overline{BC}$ , $O$ is the midpoint of $\overline{BC}$ $\therefore OB = OC$ $\therefore$ In $\triangle AOB, \triangle AOC$ $\begin{cases} OB = OC \\ \overline{AO} \text{ is common side} \\ m(\angle AOB) = m(\angle AOC) = 90^\circ \end{cases}$ $\therefore \triangle AOB \cong \triangle AOC$ $\therefore m(\angle BAO) = m(\angle CAO)$ In $\triangle AXN, \triangle AYN$ $\therefore m(\angle AXN) = m(\angle AYN) = 90^\circ$ $\therefore m(\angle XAN) = m(\angle YAN)$ $\therefore m(\angle ANX) = m(\angle ANY)$ $\therefore$ In $\triangle AXN, \triangle AYN$ $\begin{cases} m(\angle ANX) = m(\angle ANY) \\ m(\angle XAN) = m(\angle YAN) \\ \overline{AN} \text{ is a common side} \end{cases}$ $\therefore \triangle AXN \cong \triangle AYN \therefore NX = NY$ $\therefore \overline{NX} \perp \overline{BD}, \overline{NY} \perp \overline{CE}$ $\therefore BD = CE$ (Q.E.D.)
14	$\therefore \triangle MXA$ and $\triangle MYB$ which are right-angled triangles In them $\begin{cases} MA = MB \text{ (lengths of two radii)} \\ MX = MY \end{cases}$ $\therefore$ The two triangles are congruent , then we deduce that : $m(\angle MAX) = m(\angle MBY)$ $\therefore \triangle HAB$ is an isosceles triangle. (Q.E.D. 1) $\therefore \overline{MX} \perp \overline{AC}, \overline{MY} \perp \overline{BD}$ $\therefore MX = MY$ $\therefore AC = BD \therefore AH = BH$ $\therefore AH - AC = BH - BD \therefore HC = HD$ (Q.E.D. 2)
15	$\therefore X$ is the midpoint of $\overline{AB} \therefore \overline{MX} \perp \overline{AB}$ $\therefore Y$ is the midpoint of $\overline{AC} \therefore \overline{MY} \perp \overline{AC}$ $\therefore MX = MY \therefore AB = AC$ $\therefore m(\angle BAC) = 60^\circ$ $\therefore \triangle ABC$ is an equilateral triangle (Q.E.D. 1) $\therefore BM = CM = r \therefore M \in$ the axis of symmetry of $\overline{BC}$ $\therefore AB = AC \therefore A \in$ the axis of symmetry of $\overline{BC}$ $\therefore \overline{AM}$ is the axis of symmetry of $\overline{BC}$ $\therefore \overline{AM} \perp \overline{BC}$ (Q.E.D. 2)



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$\therefore X$  is the midpoint of  $\overline{AB}$   
 $\therefore \overline{MX} \perp \overline{AB}$  similarly  $\overline{MY} \perp \overline{CD}$  ,  
 $\therefore AB = CD \quad \therefore MX = MY$   
 $\therefore \triangle MYX$  is an isosceles triangle  
 $\therefore \overline{ML} \perp \overline{XY} \quad \therefore XL = LY \quad (1)$   
 $\therefore \overline{ML} \perp$  the chord  $\overline{EF} \quad \therefore EL = LF \quad (2)$   
 subtracting (1) from (2) :  $\therefore XE = YF \quad (\text{Q.E.D.})$

17

**Constr. :**  
 Draw :  $\overline{NE} \perp \overline{AB}$  ,  $\overline{NF} \perp \overline{AC}$   
 $\overline{MX} \perp \overline{AL}$  ,  $\overline{MY} \perp \overline{AK}$   
**Proof :**  $\therefore \overline{NE} \perp \overline{AB}$  ,  $\overline{NF} \perp \overline{AC}$  ,  $AB = AC$   
 $\therefore NE = NF$   
 $\therefore \triangle ANE$  and  $\triangle ANF$  which are right-angled  
 In them  $\begin{cases} NE = NF \\ \overline{AN} \text{ is a common side} \end{cases}$   
 $\therefore \triangle ANE \cong \triangle ANF$  , then we deduce that  
 $m(\angle NAE) = m(\angle NAF)$   
 $\therefore \triangle AMX$  ,  $\triangle AMY$   
 In them  $\begin{cases} \overline{AM} \text{ is common side} \\ m(\angle AXM) = m(\angle AYM) = 90^\circ \\ m(\angle XAM) = m(\angle YAM) \text{ (proved)} \end{cases}$   
 $\therefore \triangle AMX \cong \triangle AMY$  , then we deduce that  $MX = MY$   
 $\overline{MX} \perp \overline{AL}$  ,  $\overline{MY} \perp \overline{AK}$   
 $\therefore AL = AK \quad (\text{Q.E.D.})$

18

$\therefore \overline{MD} \perp \overline{AB} \quad \therefore D$  is the midpoint of  $\overline{AB} \quad (1)$   
 $\therefore \overline{ME} \perp \overline{AC} \quad \therefore E$  is the midpoint of  $\overline{AC} \quad (2)$   
 $\therefore AD = \sqrt{(2-1)^2 + (2-0)^2} = \sqrt{5}$  length units  
 $\therefore AE = \sqrt{(2-3)^2 + (2-4)^2} = \sqrt{5}$  length units  
 $\therefore AD = AE \quad \therefore AB = AC$   
 $\therefore ME = MD \quad (\text{Q.E.D.})$

19

**Constr. :**  
 Draw :  
 $\overline{NE} \perp \overline{CB}$  ,  $\overline{NF} \perp \overline{CD}$   
**Proof :**  $\therefore \overline{NE} \perp \overline{CB}$   
 $\therefore E$  is the midpoint of  $\overline{CB}$   
 In  $\triangle NEC$  which is right-angled at  $E$   
 $NE = \sqrt{(NC)^2 - (CE)^2} = \sqrt{25 - 9} = 4$  cm.  
 $\therefore NE = AM$   
 $\therefore \overline{AC}$  is a tangent to the circle  $M$  ,  $\overline{MA}$  is a radius  
 $\therefore \overline{MA} \perp \overline{AC}$

$\therefore m(\angle CEN) = m(\angle CAM) = 90^\circ$   
 and they are alternate angles  
 $\therefore \overline{NE} \parallel \overline{AM}$   
 $\therefore$  The figure  $NEAM$  is a rectangle  $\therefore \overline{NM} \parallel \overline{CA}$   
 $\therefore$  The figure  $MACN$  is a trapezium  
 Its area =  $\frac{1}{2}(MN + AC) \times AM$   
 $= \frac{1}{2}(12 + 15) \times 4 = 54 \text{ cm}^2 \quad (\text{First req.})$   
 $\therefore \overline{NF} \perp \overline{CD}$  ,  $\overline{NE} \perp \overline{CB}$  ,  $CD = CB$   
 $\therefore NF = NE = 4$  cm.  
 $\therefore$  The distance between the point  $N$  and  $\overline{CD}$  is 4 cm.  
 (Second req.)



# Final Revision

# Geometry

## Rules + Questions + Answers

### THIRD PREP

### Second Term 2019

منتري توجيه الرياضيات  
أ. عاون إدوار



## RULES OF ALGEBRA

### Basic definitions and concepts on the circle

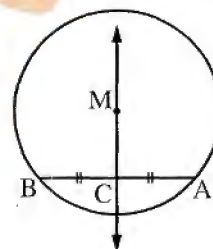
#### Corollary 1

The straight line passing through the centre of the circle and the midpoint of any chord of it (not passing through the centre) is perpendicular to this chord.

In the opposite figure :

If  $\overline{AB}$  is a chord of the circle M

and C is the midpoint of  $\overline{AB}$  , then  $\overrightarrow{MC} \perp \overline{AB}$

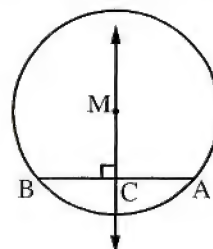


#### Corollary 2

The straight line passing through the centre of the circle and perpendicular to any chord of it bisects this chord.

In the opposite figure :

If  $\overline{AB}$  is a chord of the circle M and  $\overrightarrow{MC} \perp \overline{AB}$  , where  $C \in \overline{AB}$  , then C is the midpoint of  $\overline{AB}$



#### Corollary 3

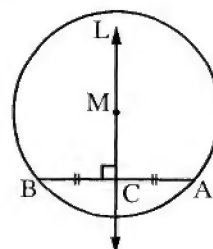
The perpendicular bisector to any chord of a circle passes through the centre of the circle.

In the opposite figure :

If  $\overline{AB}$  is a chord of the circle M , C is the midpoint of  $\overline{AB}$

and the straight line  $L \perp \overline{AB}$  from the point C ,

then  $M \in$  the straight line L



From the previous corollary , we deduce that :

The axis of symmetry of any chord of a circle passes through its centre , so this axis is also an axis of symmetry of the circle.

#### The radius of the circle

It is a line segment with one endpoint at the centre of the circle and the other endpoint on the circle.

#### The diameter of the circle

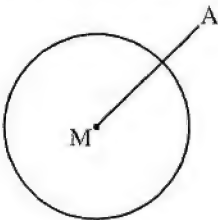
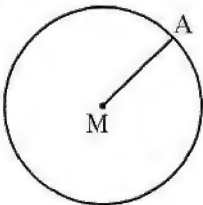
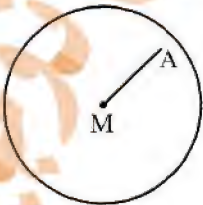
It is a chord passing through the centre of the circle.



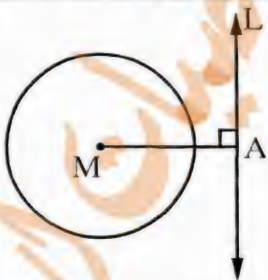
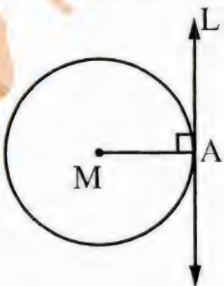
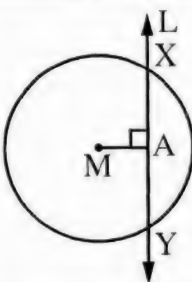
## Position of a point and a straight line With respect to a circle

### First Position of a point with respect to a given circle :

If M is a circle of radius length  $r$  and A is a point in its plane , then :

A is outside the circle M	A is on the circle M	A is inside the circle M
		
If $MA > r$	If $MA = r$	If $MA < r$

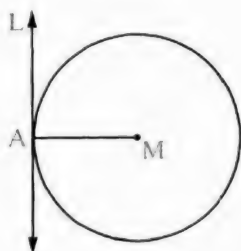
### Second Position of a straight line with respect to a circle :

If	Then	The figure	Note that
(1) $MA > r$	The straight line L lies outside the circle M		<ul style="list-style-type: none"> <li><math>L \cap \text{the circle M} = \emptyset</math></li> <li><math>L \cap \text{the surface of the circle M} = \emptyset</math></li> </ul>
(2) $MA = r$	The straight line L is a tangent to the circle M at A A is called "the point of tangency"		<ul style="list-style-type: none"> <li><math>L \cap \text{the circle M} = \{A\}</math></li> <li><math>L \cap \text{the surface of the circle M} = \{A\}</math></li> </ul>
(3) $MA < r$	The straight line L is a secant to the circle M		<ul style="list-style-type: none"> <li><math>L \cap \text{the circle M} = \{X, Y\}</math></li> <li><math>L \cap \text{the surface of the circle M} = \overline{XY}</math> <math>\overline{XY}</math> is called the chord of intersection</li> </ul>



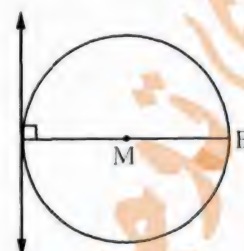
## Two important facts

- 1** The tangent to a circle is perpendicular to the radius drawn from the point of tangency.



*i.e.* if the straight line L is a tangent to the circle M at the point A ,  
then  $\overline{MA} \perp L$

- 2** The straight line which is perpendicular to the diameter of a circle at one of its endpoints is a tangent to the circle.



*i.e.* if  $\overline{AB}$  is a diameter of the circle M and the straight line  $L \perp \overline{AB}$  at the point A , then L is a tangent to the circle M at the point A

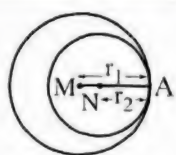
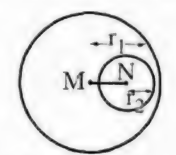
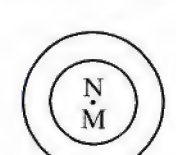
The two tangents which are drawn from the two endpoints of a diameter of a circle are parallel.

## Position of a circle with respect to another circle

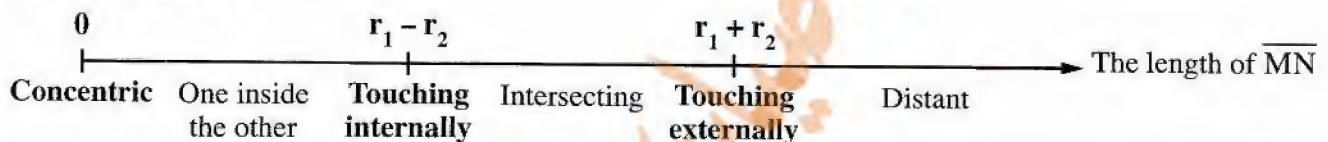
Let M and N be two circles , their radii lengths are  $r_1$  and  $r_2$  respectively ,  $r_1 > r_2$

If	Then the two circles are	Note that
<p><math>MN &gt; r_1 + r_2</math></p>	Distant	<ul style="list-style-type: none"> <li>The circle <math>M \cap</math> the circle <math>N = \emptyset</math></li> <li>The surface of circle <math>M \cap</math> the surface of circle <math>N = \emptyset</math></li> </ul>
<p><math>MN = r_1 + r_2</math></p>	Touching externally	<ul style="list-style-type: none"> <li>The circle <math>M \cap</math> the circle <math>N = \{A\}</math></li> <li>The surface of circle <math>M \cap</math> the surface of circle <math>N = \{A\}</math></li> </ul>
<p><math>r_1 - r_2 &lt; MN &lt; r_1 + r_2</math></p>	Intersecting	<ul style="list-style-type: none"> <li>The circle <math>M \cap</math> the circle <math>N = \{A, B\}</math></li> <li>The surface of circle <math>M \cap</math> the surface of circle <math>N</math> = the surface of the yellow part.</li> </ul>



 <p><math>MN = r_1 - r_2</math></p>	Touching internally	<ul style="list-style-type: none"> <li>The circle <math>M \cap</math> the circle <math>N = \{A\}</math></li> <li>The surface of circle <math>M \cap</math> the surface of circle <math>N =</math> the surface of circle <math>N</math></li> </ul>
 <p><math>MN &lt; r_1 - r_2</math></p>	One inside the other the circle N is inside the circle M	<ul style="list-style-type: none"> <li>The circle <math>M \cap</math> the circle <math>N = \emptyset</math></li> <li>The surface of circle <math>M \cap</math> the surface of circle <math>N =</math> the surface of circle <math>N</math></li> </ul>
 <p><math>MN = \text{zero}</math></p>	Concentric	<ul style="list-style-type: none"> <li>The circle <math>M \cap</math> the circle <math>N = \emptyset</math></li> <li>The surface of circle <math>M \cap</math> the surface of circle <math>N =</math> the surface of circle <math>N</math></li> </ul>

### Summary



### Notes :

From the previous summary , we notice that :

- 1 If M and N are two distant circles , then :  $MN \in ] r_1 + r_2 , \infty [$
- 2 If M and N are two intersecting circles , then :  $MN \in ] r_1 - r_2 , r_1 + r_2 [$
- 3 If M and N (one of them is inside the other) , then :  $MN \in ] 0 , r_1 - r_2 [$

### Corollary ①

The line of centres of two touching circles passes through the point of tangency and is perpendicular to the common tangent at this point.

In the two opposite figures :

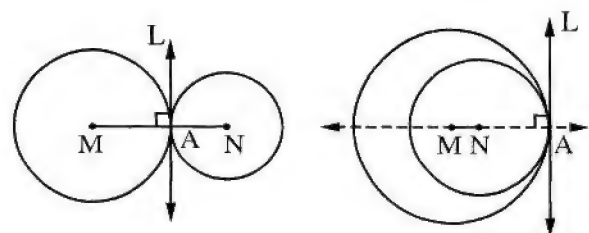
If the two circles

M and N are touching

at A (the point of tangency)

the straight line L is a common tangent to them at A

then  $A \in \overleftrightarrow{MN}$  and  $\overleftrightarrow{MN} \perp$  the straight line L





### Corollary 2

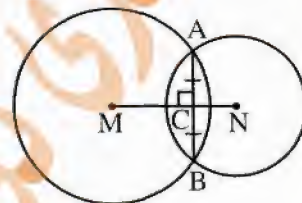
The line of centres of two intersecting circles is perpendicular to the common chord and bisects it.

In the opposite figure :

If M and N are two circles intersecting at A and B ,

then  $\overleftrightarrow{MN} \perp \overline{AB}$  ,  $\overleftrightarrow{MN}$  bisects  $\overline{AB}$  i.e.  $AC = BC$

This mean that  $\overleftrightarrow{MN}$  is the axis of symmetry of  $\overline{AB}$



## LESSON [4] Identifying the circle

We know that the circle is identified if we know :

1 its centre

2 its radius length

In the following, we will study the possibility of identifying (drawing) the circle under certain conditions.

**First** Drawing a circle passing through a given point :

*i.e.* We can draw an infinite number of circles passing through a given point.

**Second** Drawing a circle passing through two given points :

*i.e.* There is an infinite number of circles that can be drawn to pass through the two points A and B and all their centres lie on the axis of symmetry of  $\overline{AB}$

### Remarks

If  $\overline{AB}$  is a line segment and the required is drawing a circle passing through the two points A and B , then :

1 If  $r > \frac{1}{2} AB$  , then we can draw two circles (as shown in the previous example).

2 If  $r = \frac{1}{2} AB$  , then we can draw one and only one circle (it is the smallest circle) passing through the two points A and B , hence  $\overline{AB}$  is a diameter of it and its centre is the midpoint of  $\overline{AB}$

3 If  $r < \frac{1}{2} AB$  , then it is impossible to draw any circle.

• Any two circles do not intersect at more than two points.

**Third** Drawing a circle passing through three given points :

*i.e.* It is impossible to draw a circle passing through three collinear points.

For any three non-collinear points , there is a unique circle



can be drawn to pass through them.

**Notice that :**

There is a unique circle passing through three points as A , B and C which are not collinear and the centre of this circle is the point of intersection of any two axes of symmetry of the axes of the line segments  $\overline{AB}$  ,  $\overline{BC}$  and  $\overline{AC}$

**Corollary 1**

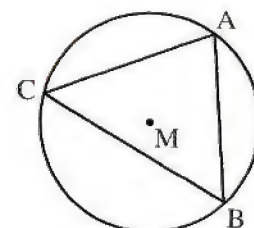
The circle which passes through the vertices of a triangle is called the circumcircle of this triangle.

- The triangle whose vertices lie on a circle is called the inscribed triangle of this circle.

**In the opposite figure :**

M is the circumcircle of  $\triangle ABC$

or  $\triangle ABC$  is the inscribed triangle of the circle M



**Corollary 2**

The perpendicular bisectors of the sides of a triangle intersect at a point which is the centre of the circumcircle of the triangle.

**In the opposite figure :**

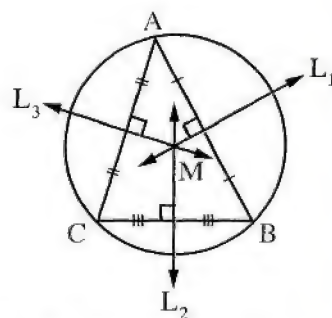
If the straight lines  $L_1$  ,  $L_2$  and  $L_3$  are the axes

of  $\overline{AB}$  ,  $\overline{BC}$  and  $\overline{CA}$  respectively

and  $L_1 \cap L_2 \cap L_3 = \{M\}$  ,

then the point M is the centre of the circumcircle of  $\triangle ABC$

**The position of the centre of the circumcircle of the triangle as M differs according to the type of the triangle as shown in the following table :**



The acute-angled triangle	The right-angled triangle	The obtuse-angled triangle
M is <b>inside</b> the triangle	M is the <b>midpoint of the hypotenuse</b>	M is <b>outside</b> the triangle



### A special case :

The centre of the circumcircle of an equilateral triangle is :

- The point of intersection of its sides axes.
- The point of intersection of its altitudes.
- The point of intersection of its medians.
- The point of intersection of the bisectors of its angles.



### Notice that :

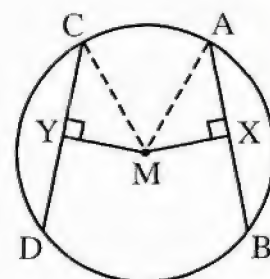
We can draw a circle passing through the vertices of (a rectangle or a square or an isosceles trapezium) while we cannot draw a circle passing through the vertices of (the parallelogram or the rhombus or the trapezium which is not isosceles).

## The relation between the chords of a circle and its center

### Theorem

If chords of a circle are equal in length, then they are equidistant from the centre.

Given	$AB = CD$ , $\overline{MX} \perp \overline{AB}$ and $\overline{MY} \perp \overline{CD}$
R.T.P.	$MX = MY$
Construction	Draw $\overline{MA}$ and $\overline{MC}$



### Corollary

In congruent circles, chords which are equal in length are equidistant from the centres.

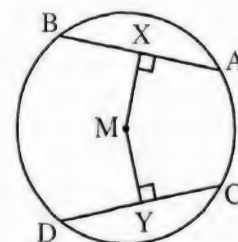
Converse of the theorem (without proof) :

In the same circle (or in congruent circles) ,  
chords which are equidistant from the centre (s) are equal in length.

*i.e.* In the opposite figure :

If  $\overline{AB}$  and  $\overline{CD}$  are two chords of the circle M ,

$\overline{MX} \perp \overline{AB}$  ,  $\overline{MY} \perp \overline{CD}$  and  $MX = MY$  , then  $AB = CD$





## Central angles and measuring arcs

i.e.

The measure of the semicircle =  $180^\circ$  and then  
the measure of the circle =  $2 \times 180^\circ = 360^\circ$

### Remark

The two adjacent arcs are two arcs in the same circle that have only one point in common.

### The length of the arc

It is a part of a circle's circumference proportional to its measure and it is measured by length units (centimetre , metre , ...)

- To calculate the length of the arc, you can use the following rule :

$$\begin{aligned} \text{The length of the arc} &= \frac{\text{the measure of the arc}}{\text{the measure of the circle}} \times \text{the circumference of the circle} \\ &= \frac{\text{the measure of the arc}}{360^\circ} \times 2 \pi r \end{aligned}$$

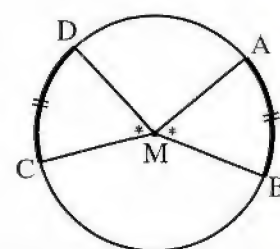
Where  $r$  is the radius length of the circle and  $\pi$  is the approximated ratio.

### Corollary 1

In the same circle (or in congruent circles) , if the measures of arcs are equal , then the lengths of the arcs are equal and vice versa.

In the opposite figure :

If  $M$  is a circle in which  $m(\widehat{AB}) = m(\widehat{CD})$   
 , then the length of  $\widehat{AB}$  = the length of  $\widehat{CD}$   
 and vice versa if the length of  $\widehat{AB}$   
 = the length of  $\widehat{CD}$  , then  $m(\widehat{AB}) = m(\widehat{CD})$

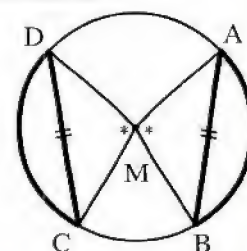


### Corollary 2

In the same circle (or in congruent circles) , if the measures of arcs are equal , then their chords are equal in length , and vice versa.

In the opposite figure :

If  $M$  is a circle in which  
 $m(\widehat{AB}) = m(\widehat{CD})$  , then  $AB = CD$  and vice versa  
 If  $AB = CD$  , then  $m(\widehat{AB}) = m(\widehat{CD})$



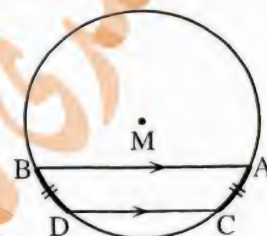


### Corollary 3

If two parallel chords are drawn in a circle , then the measures of the two arcs between them are equal.

In the opposite figure :

If  $\overline{AB}$  and  $\overline{CD}$  are two chords in the circle M  
 $\overline{AB} \parallel \overline{CD}$  , then  $m(\widehat{AC}) = m(\widehat{BD})$

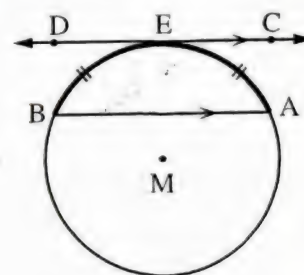


### Corollary 4

If a chord is parallel to a tangent of a circle , then the measures of the two arcs between them are equal.

In the opposite figure :

If  $\overline{AB}$  is a chord in the circle M and  
 $\overline{CD}$  touches the circle M at E ,  
 $\overline{CD} \parallel \overline{AB}$  , then  $m(\widehat{EA}) = m(\widehat{EB})$



## The relation between the inscribed and Central angles subtended by the same arc theorem

### The inscribed angle

It is the angle whose vertex lies on the circle and its sides contain two chords of the circle.

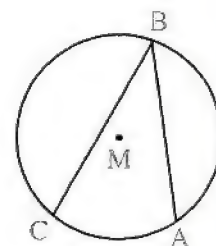
In the opposite figure :

- $\angle ABC$  is an inscribed angle

because its vertex B belongs to the circle M

and its sides  $\overrightarrow{BA}$  and  $\overrightarrow{BC}$  carry the two chords  $\overline{BA}$  and  $\overline{BC}$  in the circle M

- The inscribed angle  $\angle ABC$  is subtended by  $\widehat{AC}$



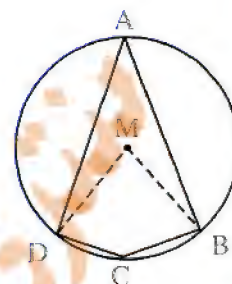
### Remark

For each inscribed angle , there is one central angle subtended by the same arc.



**In the opposite figure :**

- The inscribed angle  $\angle BAD$  is subtended with the central angle  $\angle BMD$  by the arc  $\widehat{BD}$
- While the inscribed angle  $\angle BCD$  is subtended with the reflex central angle  $BMD$  by the arc  $\widehat{BAD}$



### Theorem (1)

**The measure of the inscribed angle is half the measure of the central angle , subtended by the same arc.**

#### Remark

The measure of the central angle equals twice the measure of the inscribed angle subtended by the same arc.

## The corollaries of theorem (1) and its well known problems

### Corollary 1

**The measure of an inscribed angle is half the measure of the subtended arc.**

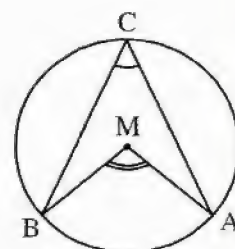
**In the opposite figure :**

$$m(\angle C) = \frac{1}{2} m(\angle AMB)$$

(inscribed and central angles with common arc  $\widehat{AB}$ ),

$$m(\angle AMB) = m(\widehat{AB})$$

$$\therefore m(\angle C) = \frac{1}{2} m(\widehat{AB})$$



#### Remark

The measure of the arc equals twice the measure of the inscribed angle subtended by this arc.

### Corollary 2

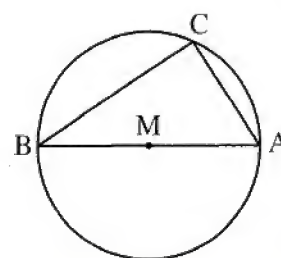
**The inscribed angle in a semicircle is a right angle.**

**In the opposite figure :**

$$\therefore m(\angle C) = \frac{1}{2} m(\widehat{AB}) \text{ (corollary 1) ,}$$

$$\therefore m(\widehat{AB}) = 180^\circ$$

$$\therefore m(\angle C) = 90^\circ$$





### Remarks

- 1 The inscribed angle which is right angle is drawn in a semicircle.
- 2 The inscribed angle which is subtended by an arc of measure less than the measure of a semicircle is an acute angle.
- 3 The inscribed angle which is subtended by an arc of measure greater than the measure of a semicircle is an obtuse angle.

### Well known problem (1)

If two chords intersect at a point inside a circle, then the measure of the included angle equals half of the sum of the two measures of the two opposite arcs.

Given

$\overline{AB}$ ,  $\overline{CD}$  are two chords in a circle intersecting at the point E

R.T.P.

$$1 \quad m(\angle AEC) = \frac{1}{2} [m(\widehat{AC}) + m(\widehat{BD})]$$

$$2 \quad m(\angle CEB) = \frac{1}{2} [m(\widehat{BC}) + m(\widehat{AD})]$$

### Well known problem (2)

If two rays carrying two chords in a circle are intersecting outside it, then the measure of their intersecting angle equals half of the measure of the major arc subtracted from it half of the measure of the minor arc in which both are included by the two sides of this angle.

Given

$$\overrightarrow{CB} \cap \overrightarrow{ED} = \{A\}$$

R.T.P.

$$m(\angle A) = \frac{1}{2} [m(\widehat{CE}) - m(\widehat{BD})]$$

## Inscribed angles subtended by same arc Theorem (2) its corollaries

### Theorem 2

In the same circle, the measures of all inscribed angles subtended by the same arc are equal.

Given

$\angle C$ ,  $\angle D$  and  $\angle E$  are inscribed angles subtended by  $\widehat{AB}$

R.T.P.

$$m(\angle C) = m(\angle D) = m(\angle E)$$

Proof

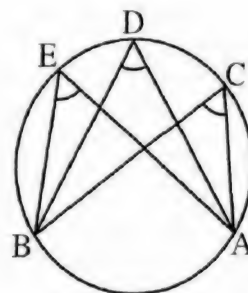
$$\therefore m(\angle C) = \frac{1}{2} m(\widehat{AB})$$

$$, m(\angle D) = \frac{1}{2} m(\widehat{AB})$$

$$, m(\angle E) = \frac{1}{2} m(\widehat{AB})$$

$$\therefore m(\angle C) = m(\angle D) = m(\angle E)$$

(Q.E.D.)





### Corollary

In the same circle (or in any number of circles) the measures of the inscribed angles subtended by arcs of equal measures are equal.

i.e. In the circle M

If  $m(\widehat{AB}) = m(\widehat{CD})$ ,

then  $m(\angle X) = m(\angle Y)$



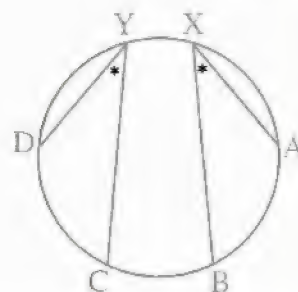
The converse of the previous corollary is true also

i.e. In the same circle (or in any number of circles) the inscribed angles of equal measures subtend arcs of equal measures.

In the opposite figure :

If  $m(\angle X) = m(\angle Y)$ ,

then  $m(\widehat{AB}) = m(\widehat{CD})$



## The cyclic quadrilateral-the converse of theorem (2)

The cyclic quadrilateral :

It is a quadrilateral figure whose four vertices belong to one circle.

### The converse of theorem (2) (without proof)

If two angles subtended by the same base and on the same side of it have the same measure, then their vertices are on an arc of a circle and the base is a chord of it.

### Remarks

- 1 If there are two angles drawn on one of the sides of a quadrilateral, they are on the same side of it and they are not equal in measure, then the quadrilateral is not cyclic.
- 2 Each of the rectangle, the square and the isosceles trapezium are cyclic quadrilaterals while each of the parallelogram, the rhombus and the trapezium that is not isosceles are not cyclic quadrilaterals.



## Properties of The cyclic quadrilateral theorem (3)

### Theorem 3

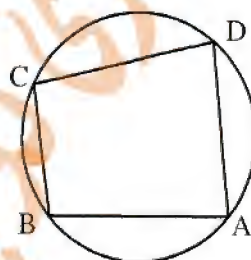
In a cyclic quadrilateral, each two opposite angles are supplementary.

**Given** ABCD is a cyclic quadrilateral

**R.T.P.** **1**  $m(\angle A) + m(\angle C) = 180^\circ$

**2**  $m(\angle B) + m(\angle D) = 180^\circ$

**Proof**  $\therefore m(\angle A) = \frac{1}{2} m(\widehat{BCD})$  and  $m(\angle C) = \frac{1}{2} m(\widehat{BAD})$   
 $\therefore m(\angle A) + m(\angle C) = \frac{1}{2} [m(\widehat{BCD}) + m(\widehat{BAD})]$   
 $= \frac{1}{2} \text{ the measure of the circle} = \frac{1}{2} \times 360^\circ = 180^\circ$



**Similarly:**  $m(\angle B) + m(\angle D) = 180^\circ$  (Q.E.D.)

### Corollary

The measure of the exterior angle at a vertex of a cyclic quadrilateral is equal to the measure of the interior angle at the opposite vertex.

In the opposite figure :

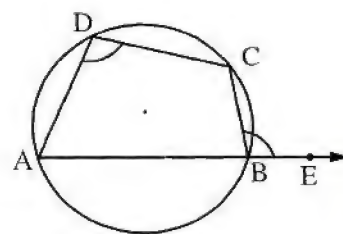
If ABCD is a cyclic quadrilateral

,  $\angle CBE$  is an exterior angle of it ,

then  $m(\angle ABC) + m(\angle D) = 180^\circ$

but  $m(\angle ABC) + m(\angle CBE) = 180^\circ$

$\therefore m(\angle CBE) = m(\angle D)$



## The converse of theorem (3) and its corollary

A summary of the cases in which the quadrilateral is cyclic :

The quadrilateral is cyclic if one of the following conditions is verified :

- 1** If there is a point in the plane of the figure such that it is equidistant from its vertices.
- 2** If there are two equal angles in measure and drawn on one of its sides as a base and on one side of this side.
- 3** If there are two opposite supplementary angles «their sum =  $180^\circ$ »



- 4** If there is an exterior angle at any of its vertices equal in measure to the measure of the interior angle at the opposite vertex.

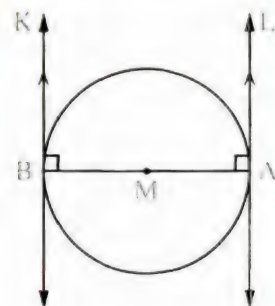
## The relation between the tangents of a Circle theorem (4) and its corollaries

**First**

**The two tangents drawn at the two ends of a diameter in a circle are parallel.**

**i.e. In the opposite figure :**

If  $\overline{AB}$  is a diameter in the circle M and the two straight lines L and K are two tangents to the circle at A and B respectively ,  
then the straight line L // the straight line K  
(because the straight line L  $\perp$   $\overline{AB}$  and the straight line K  $\perp$   $\overline{AB}$ )

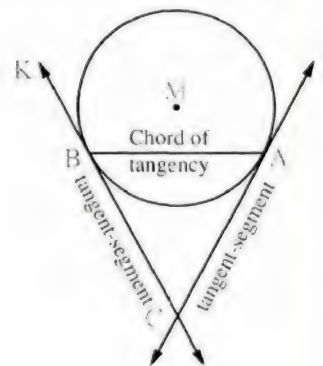


**Second**

**The two tangents drawn at the two ends of a chord of a circle are intersecting.**

**i.e. In the opposite figure :**

If  $\overline{AB}$  is a chord in the circle M and the two straight lines L and K are two tangents to the circle at A and B respectively , then the two straight lines L and K are intersecting at a point outside the circle M (Say C) and  $\overline{AC}$  ,  $\overline{BC}$  are called tangent - segments and  $\overline{AB}$  is called a chord of tangency.



### Theorem (4)

**The two tangent-segments drawn to a circle from a point outside it are equal in length.**

### Corollaries of theorem (4) :

#### Corollary ①

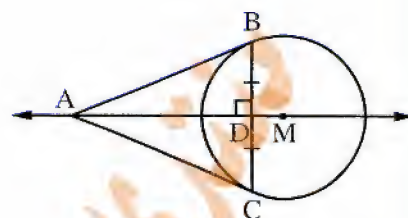
**The straight line passing through the centre of the circle and the intersection point of the two tangents is an axis of symmetry to the chord of tangency of those two tangents.**



**In the opposite figure :**

If  $\overrightarrow{AB}$  and  $\overrightarrow{AC}$  are two tangents to the circle M at B and C respectively, then  $\overrightarrow{AM}$  is the axis of symmetry to  $\overline{BC}$

**i.e.**  $\overrightarrow{AM} \perp \overline{BC}$  ,  $BD = CD$



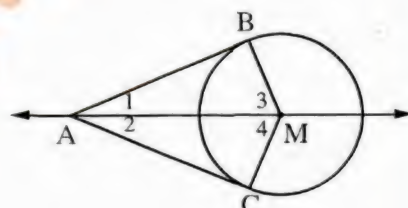
### Corollary 2

The straight line passing through the centre of the circle and the intersection point of its two tangents bisects the angle between these two tangents. It also bisects the angle between the two radii passing through the two points of tangency.

**In the opposite figure :**

If  $\overrightarrow{AB}$  and  $\overrightarrow{AC}$  are two tangents to the circle M at B and C respectively then :

- $\overrightarrow{AM}$  bisects  $\angle BAC$   $\therefore m(\angle 1) = m(\angle 2)$
- $\overrightarrow{MA}$  bisects  $\angle BMC$   $\therefore m(\angle 3) = m(\angle 4)$



### Remarks on theorem (4) and its corollaries

**In the opposite figure :**

**1**  $AB = AC$  **2**  $MB = MC = r$

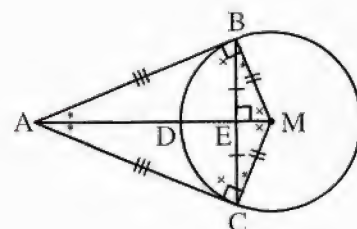
**3**  $BE = CE$  ,  $\overrightarrow{AM} \perp \overline{BC}$

**4**  $m(\angle ABM) = m(\angle ACM) = 90^\circ$

**i.e.** The figure ABMC is a cyclic quadrilateral.

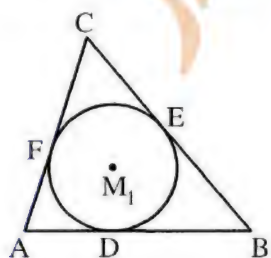
**5**  $m(\angle BAM) = m(\angle BCM) = m(\angle CAM) = m(\angle CBM)$

**6**  $m(\angle AMB) = m(\angle ACB) = m(\angle AMC) = m(\angle ABC)$

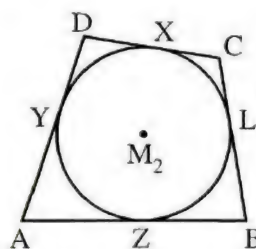


### Definition

The inscribed circle of a polygon is the circle which touches all of its sides internally.



**Fig. (1)**



**Fig. (2)**

**In figure (1) :**  $M_1$  is the inscribed circle of the triangle ABC where :

the side  $\overline{AB}$  touches the circle at D , the side  $\overline{BC}$  touches the circle at E



and the side  $\overline{CA}$  touches the circle at F.

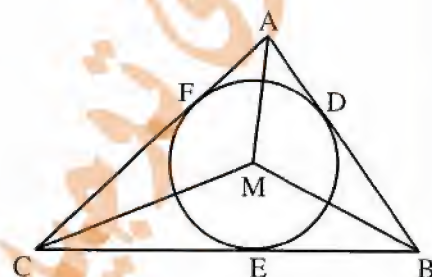
**In figure (2) :**  $M_2$  is the inscribed circle of the quadrilateral ABCD

**Remark**

The centre of the inscribed circle of any triangle is the point of intersection of the bisectors of its interior angles.

**i.e. In the opposite figure :**

If the circle M is the inscribed circle of the triangle ABC then M is the intersection point of the bisectors of the interior angles of  $\triangle ABC$



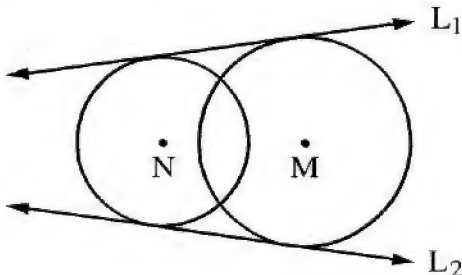
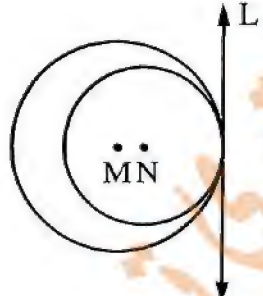

**The common tangents to two circles**

- It is said that the tangent  $\overline{AB}$  is an internal common tangent to the two circles M and N if the two circles M and N are on two different sides of the tangent.
- It is said that the tangent  $\overline{AB}$  is an external common tangent of the two circles M and N if the two circles M and N are on the same side of the tangent.

*The following table shows the number of the common tangents to two circles in their different situations (locations) :*

Two distant circles	Two circles touching externally
<p><b>4 common tangents</b></p> <ul style="list-style-type: none"> <li>• <math>L_1</math> and <math>L_2</math> (external)</li> <li>• <math>L_3</math> and <math>L_4</math> (internal)</li> </ul>	<p><b>3 common tangents</b></p> <ul style="list-style-type: none"> <li>• <math>L_1</math> and <math>L_2</math> (external)</li> <li>• <math>L_3</math> (internal)</li> </ul>



Two intersecting circles	Two circles touching internally
 <p><b>2 common tangents</b></p> <ul style="list-style-type: none"> <li>• <math>L_1</math> and <math>L_2</math> (external)</li> <li>• There are no internal tangents</li> </ul>	 <p><b>One common tangent</b></p> <ul style="list-style-type: none"> <li>• L is the common tangent (external)</li> <li>• There are no internal tangents</li> </ul>
One circle inside the other	
 <p><b>There are no common tangents</b></p>	

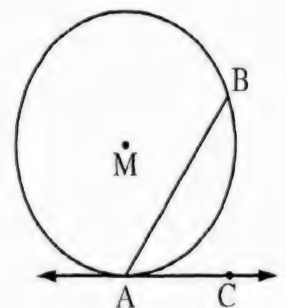
## Angles of tangency theorem (5) and its corollaries

### Definition

The angle of tangency is the angle which is composed of the union of two rays , one of them is a tangent to the circle and the other contains a chord of the circle passing through the point of tangency.

In the opposite figure :

If  $\overrightarrow{AC}$  is a tangent to the circle at A and  $\overrightarrow{AB}$  contains the chord  $\overline{AB}$  , then  $\angle BAC$  is an angle of tangency in the circle M , its chord is  $\overline{AB}$   $\overline{AB}$  is called the chord of tangency of the angle of tangency  $\angle BAC$



i.e. The measure of the angle of tangency =  $\frac{1}{2}$  the measure of the arc intercepted by its sides.



### Theorem 5

The measure of the angle of tangency is equal to the measure of the inscribed angle subtended by the same arc.

**Given**  $\angle BAC$  is an angle of tangency and  $\angle D$  is an inscribed angle.

**R.T.P.**  $m(\angle BAC) = m(\angle D)$

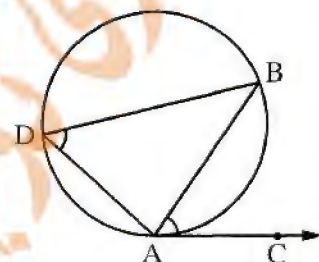
**Proof**  $\because \angle BAC$  is an angle of tangency.

$$\therefore m(\angle BAC) = \frac{1}{2} m(\widehat{AB}) \quad (1)$$

$\because \angle D$  is an inscribed angle

$$\therefore m(\angle D) = \frac{1}{2} m(\widehat{AB}) \quad (2)$$

From (1) and (2), we deduce that :  $m(\angle BAC) = m(\angle D)$  (Q.E.D.)



### Corollary

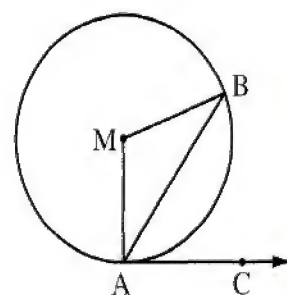
The measure of the angle of tangency is half the measure of the central angle subtended by the same arc.

In the opposite figure :

$$m(\angle BAC) \text{ (tangency angle)} = \frac{1}{2} m(\widehat{AB})$$

$$\because m(\angle AMB) \text{ (central angle)} = m(\widehat{AB})$$

$$\therefore m(\angle BAC) \text{ (tangency angle)} = \frac{1}{2} m(\angle AMB) \text{ (central angle)}$$



### Remark

The angle of tangency is supplementary to the drawn inscribed angle on the chord of the angle of tangency and in one side of it.

## The converse of theorem (5)

If a ray is drawn from one end of a chord of a circle so that the angle between this ray and the chord is equal in measure to the inscribed angle subtended by the chord in the alternate side, then this ray is a tangent to the circle.



# Questions Part (1)

## Geometry

### General Exercise on Unit One

**First: Complete the following**

- 1) If one end of a line segment lies on the center of the circle and the other end on the circle, then this line segment is called .....
- 2) If the two ends of a line segment lie on the circle, then this line segment is called .....
- 3) The chord which passes through the center of the circle is called .....
- 4) The longest chord of the circle is called .....
- 5) The circle has ..... number of axes of symmetry.
- 6) In any circle the perpendicular straight line on any chord from its mid-point is ..... to the circle.
- 7) The circle divides the plane into ..... sets of points.
- 8) The perpendicular straight line on the diameter from one end is .....
- 9) The two tangents to a circle at the two end points of the diameter are .....
- 10) The equal chords in length of a circle are equidistant from .....
- 11) The chords of a circle are equidistant from its center are .....
- 12) If the point A lies outside the circle M of radius, then  $MA$  .....  $R$ .
- 13) The line of centers of two intersecting circles is ..... , .....
- 14) If the surface of the circle  $M \cap$  the surface of the circle  $N = \varnothing$ , then the two circles M and N are.....
- 15) If the surface of the circle  $M \cap$  the surface of the circle  $N = \{A\}$ , then the two circles M and N are .....
- 16) The number of circles can be drawn passing through two given points in the plane equals .....
- 17) If two circles have three common points, then they are .....
- 18) The radius of the smallest circle drawn to pass through two given points in the plane equals .....
- 19) The point of intersection of the symmetric axes of the sides of a triangle is .....
- 20) If M is a circle of radius  $r$ , A is a point in the plane of the circle:
  - (a) If  $MA = \frac{1}{2} R$ , then A ..... the circle
  - (b) If  $MA = R$ , then A ..... the circle
  - (c) If  $MA = 3 R$ , then A ..... the circle



**Second: Match from the column (X) to the column (Y) to get a true statement**

Two circles of radii 8 cm. & 6 cm.

X	Y
1) If $MN = 1$ cm	a) M , N are two intersecting circles
2) If $MN = 2$ cm	b) M , N are two distant circles
3) If $MN = 7$ cm	c) M , N touching externally
4) If $MN = 14$ cm	d) M , N are two interior circles
5) If $MN = 15$ cm	e) M , N touching internally

**Third : Choose the correct from the given ones :**

- If the length of a diameter of a circle is 7 cm, and the straight line L at distant 3.5 cm from its center, then L is .....  
 a) Secant to the circle at two points      b) Lies outside the circle.  
 c) Tangent to the circle      d) Axis of symmetry to the circle
- If the point A belongs to the circle M of diameter 6 cm, then MA equals  
 a) 3 cm      b) 4 cm      c) 5 cm      d) 6 cm
- If the straight line L is a tangent to the circle M of diameter 8 cm, then the distance between L and its center equals .....  
 a) 3 cm      b) 4 cm      c) 6 cm      d) 8 cm
- If the straight line L is outside a circle of radius 3 cm and its center M, If L at distance X from its center, then  $x \in$  .....  
 a)  $]3, \infty[$       b)  $[3, \infty[$       c)  $]6, \infty[$       d)  $] - \infty, - 6[$
- If the straight line L at distance x from a circle of center M and radius R,  $x \in ]0, R[$ , then L .....  
 a) Intersects the circle      b) Touches the circle  
 c) Lies outside the circle      d) Passes through the center of the circle
- If the length of the perpendicular drawn from the center of the circle on the straight line L equals 6 cm and the radius 6 cm, then L .....  
 a) Intersects the circle      b) Touches the circle  
 c) Lies outside the circle      d) Passes through the center of the circle



- [illegible]





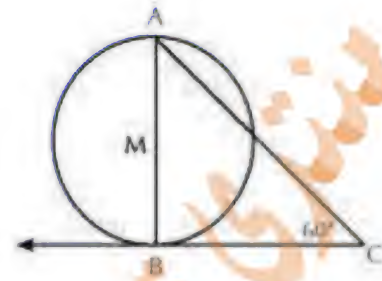


- 3) In the opposite figure

A circle of circumference 44 cm,  $\overline{AB}$  is a diameter

$\overrightarrow{BC}$  is a tangent at B, and  $m(\angle C) = 60^\circ$ .

Find the length of  $\overline{BC}$  ( $\pi = \frac{22}{7}$ )

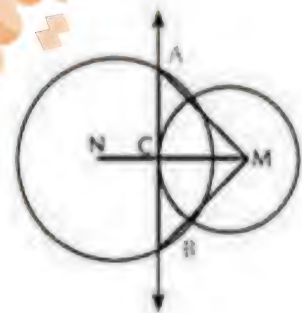


- 4) In the opposite figure

M, N are two intersecting circles,  $\overline{MN}$  intersects the circle M at C,  $\overline{CA}$  is a tangent to the circle M at C, and cuts the circle N at A, B. Prove that :

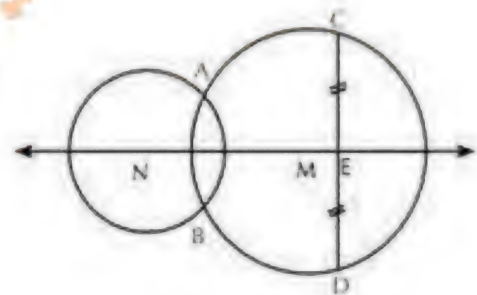
[1]  $CA = CB$

[2]  $MA = MB$



- 5) In the opposite figure

M, N are two intersecting circles,  $\overline{CD}$  is a chord in the circle M, cuts  $\overline{MN}$  at E, if E is the mid point of  $\overline{CD}$  Prove that :  $\overline{AB} \parallel \overline{CD}$



- 6) M, N are two touching internally circles at A, the circle M is greater than the circle N, draw the common tangent  $\overline{AC}$ , then draw  $\overline{NM}$  to cut the circle N at B, and draw the tangent  $\overline{BD}$  to the circle N to cut the circle M at D, E. Prove that:

[1]  $\overline{AC} \parallel \overline{BD}$

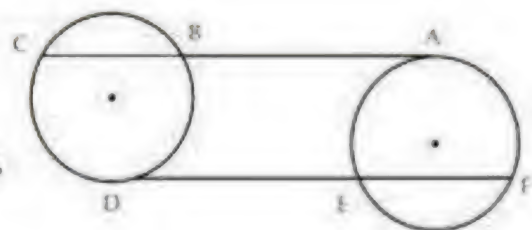
[2]  $BD = BE$ .

- 7) In the opposite figure

M, N are two congruent circles,  $\overline{AC}$  is a common tangent to the circle M at A the,  $\overline{DF}$  is a common tangent to the circle N at D,  $\overline{AC} \parallel \overline{DF}$ . Prove that :

[1]  $BC = FE$

[2]  $AB = ED$





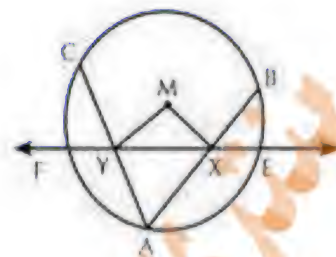
8) In the opposite figure

$\overline{AB}$ ,  $\overline{CD}$  are two chords (equal in length) in a circle M.

If X, Y are the two mid points of  $\overline{AB}$ ,  $\overline{CD}$  respectively,

$\overleftrightarrow{XY}$  cuts the circle at E and F. Prove that :

$XE = YF$ .



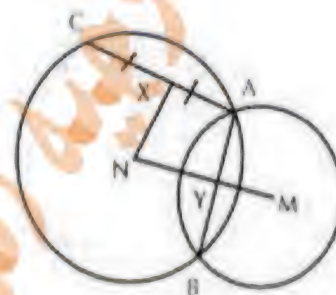
9) In the opposite figure

M, N are two intersecting circles at A, B,

$\overleftrightarrow{MN} \cap \overline{AB} = \{Y\}$ ,  $AB = AC$ , if

X is the mid point of  $\overline{AC}$ .

Prove that :  $NX = NY$ .



10)  $\overline{AB}$ ,  $\overline{CD}$  are two parallel chords in a circle M, E is the midpoint of  $\overline{AB}$ ,  $\overleftrightarrow{EM}$  is drawn to cut  $\overline{CD}$  at F. Prove that :  $FC = FD$ .

11)  $\overline{AB}$ ,  $\overline{AC}$  are two chords in a circle M, if D, E are the two the mid points of  $\overline{AB}$ ,  $\overline{AC}$  respectively,  $\overleftrightarrow{DM}$  is drawn to cut  $\overline{AC}$  at F such  $ME = EF$ . Prove that:  $m(\angle BAC) = 45^\circ$ .

12)  $\overline{AB}$  is a diameter in a circle M, the chord  $\overline{CD}$  is drawn such that  $\overline{CD} \parallel \overline{AB}$ ,  $\overline{CX} \perp \overline{AB}$  and  $\overline{DY} \perp \overline{AB}$  Prove that :  $AX = BY$ .

13) A, B are two points where  $AB = 6$  cm, Draw a circle of radius 5 cm and passes through the two points A, B. Find the distance from the center to  $\overline{AB}$ .

14) Draw the triangle ABC in which  $AB = 6$  cm,  $AC = 4$  cm,  $m(\angle BAC) = 60^\circ$ . Then draw a circle passes through the two points A, C and its center  $\in \overline{AB}$ .

15)  $\overline{AB}$  is a diameter in a circle M,  $\overline{AC}$  is a chord such that  $m(\angle BAC) = 30^\circ$ , then draw  $\overline{BC}$  and  $\overline{MD} \perp \overline{AC}$  to cut it at D. Prove that :

[1]  $\overline{MD} \parallel \overline{BC}$

[2]  $BC =$  the length of the radius of this circle.



## General Exercise on the Second Unit

### First : Complete the following

- (1) The two opposite angles in the cyclic quadrilateral are .....
- (2) The chords which opposite to equal arcs in any circle are .....
- (3) The measure of the inscribed angle equals half the measure of .....

- (4) In the opposite figure

In a circle M,  $m(\angle A) = 48^\circ$ , then:

[1]  $m(\angle C) = \dots\dots\dots$

[2]  $m(\widehat{BD}) = \dots\dots\dots$  "  $\widehat{BD}$  is the major arc"



- (5) The quadrilateral is said to be a cyclic quad. If the measure of an exterior angle at any vertex equals the ..... of the angle which opposite to its adjacent.

- (6) In the opposite figure

In a circle M,  $m(\angle CAE) = 36^\circ$ , then:

(a)  $m(\angle EBC) = \dots\dots\dots$

(b)  $m(\angle EMC) = \dots\dots\dots$

(c)  $m(\angle EDC) = \dots\dots\dots$



- (7) The inscribed angle which opposite to a minor arc in a circle is .....
- (8) The two parallel chords in a circle intercept two ..... arcs
- (9) The measure of an arc of a circle equals double .....

### Second : Choose the correct answer from the given ones

- 1) The inscribed angle which opposite to the minor arc in a circle is .....

(a) reflex

(b) right

(c) obtuse

(d) acute

- 2) In the opposite figure

In a circle M,  $m(\angle AMB) = 52^\circ$ , then

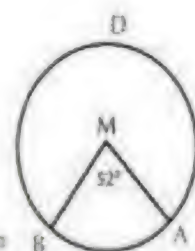
$m(\angle ADB) = \dots\dots\dots^\circ$

(a)  $52^\circ$

(b)  $104^\circ$

(c)  $128^\circ$

(d)  $308^\circ$





- 3) In the opposite figure

$\overline{AB}$  is a diameter in a circle M,

$m(\angle ABC) = 40^\circ$ , then  $m(\widehat{BC}) = \dots\dots^\circ$

- (a)  $40^\circ$  (b)  $50^\circ$  (c)  $90^\circ$  (d)  $100^\circ$



- 4) In the opposite figure

$\overline{AB}$  is a diameter in a circle M,

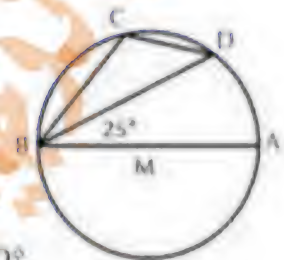
$m(\angle ABD) = 25^\circ$ , then

[1]  $m(\angle DAB) = \dots\dots^\circ$

- (a)  $25^\circ$  (b)  $50^\circ$  (c)  $65^\circ$  (d)  $90^\circ$

[2]  $m(\angle DCB) = \dots\dots^\circ$

- (a)  $50^\circ$  (b)  $100^\circ$  (c)  $115^\circ$  (d)  $125^\circ$

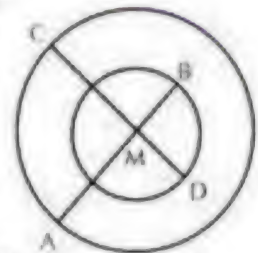


- 5) In the opposite figure

Two concentric circles at M,  $\overline{AB} \cap \overline{CD} = \{M\}$ ,

if  $m(\widehat{BD}) = 80^\circ$ , then  $m(\widehat{AC}) = \dots\dots^\circ$

- (a)  $40^\circ$  (b)  $80^\circ$  (c)  $100^\circ$  (d)  $160^\circ$



- 6) Using the following figures choose the correct answer



Figure (1)

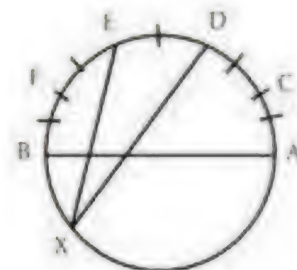


Figure (2)

In Figure (1) : A circle of center M,  $m(\angle MBC) = 32^\circ$ , then  $m(\widehat{BC}) = \dots\dots^\circ$

- (a)  $16^\circ$  (b)  $32^\circ$  (c)  $64^\circ$  (d)  $116^\circ$

In Figure (2) :  $\overline{AB}$  is a diameter in a circle,

$m(\widehat{AC}) = m(\widehat{CD}) = m(\widehat{DE}) = m(\widehat{EF}) = m(\widehat{FB})$ , then  $m(\angle DXE) = \dots\dots^\circ$

- (a)  $18^\circ$  (b)  $36^\circ$  (c)  $54^\circ$  (d)  $72^\circ$



## Model Answers Part (1)

### First Complete:

- (1) radius      (2) chord      (3) diameter      (4) diameter  
 (5) infinite      (6) axis of symmetry      (7) three  
 (8) tangent      (9) parallel      (10) its centre  
 (11) equal in length      (12) >  
 (13) perpendicular to the common chord and bisect it  
 (14) distant      (15) touching externally  
 (16) infinite      (17) congruent (coincide)  
 (18) half the length of the line segment joining the two points.  
 (19) the centre of the circumcircle.  
 (20) inside – lies on – outside.

### Second match:

- (1) (d)      (2) (e)      (3) (a)      (4) (c)      (5) (b)

### Third choose :

- (1) c      (2) a      (3) b      (4) a      (5) a  
 (6) b      (7) d      (8) c      (9) d      (10) d  
 (11) b      (12) c      (13) c      (14) a      (15) c  
 (16) a      (17) b      (18) d      (19) a      (20) c

### Fourth Answer the following questions:

- (1)  $\because \overline{MD} \perp \overline{AC}$  ,  $\because \overline{MH} \perp \overline{AB}$   
 $\therefore D, H$  are mid points of  $\overline{AC}$  and  $\overline{AB}$  respectively  
 $\therefore DH = \frac{1}{2}CB = 8 \div 2 = 4 \text{ cm.}$



(2)  $\therefore C$  is a mid point of  $\overline{AB}$

$\therefore \overline{MC} \perp \overline{AB}$

In  $\triangle AMC$

$$MC^2 = AM^2 - AC^2 = 169 - 144 = 25$$

$$MC = 5 \text{ cm} \quad , \quad \therefore MD = 13 \text{ cm}$$

$$\therefore CD = 13 - 5 = 8 \text{ cm.}$$

$$\text{area of } \triangle ADB = \frac{1}{2} \times AB \times DC$$

$$= \frac{1}{2} \times 24 \times 8 = 96 \text{ cm}^2$$

(3)  $\text{cir.} = \pi \times D \Rightarrow 44 = \frac{22}{7} \times D$

$$\therefore D = 14 \text{ cm}$$

$\therefore \overline{BC}$  is a tangent.

$$\therefore m(\angle ABC) = 90^\circ, \quad \therefore m(\angle A) = 30^\circ$$

$$\therefore BC = \frac{1}{2} AC$$

Let  $BC = x$  ,  $AC = 2x$

$$AC^2 = AB^2 + BC^2$$

$$(2x)^2 = (14)^2 + (x)^2$$

$$4x^2 = 196 + x^2$$

$$3x^2 = 196 \quad \rightarrow x^2 \simeq 65.33$$

$$\therefore x = \sqrt{65.33} \simeq 8.08 \text{ cm} = BC$$

(4)  $\therefore \overline{CA}$  and  $\overline{CB}$  are two tangents

$$\therefore \overline{MA} \perp \overline{AC} \quad , \quad \overline{MB} \perp \overline{BC}$$

$$\therefore m(\angle A) + m(\angle B) = 180^\circ$$

$\therefore AMBC$  is acyclic quad.

$$\therefore m(\angle DMB) = m(\angle ACB)$$

Exterior = opposite interior



(4) In circle M  $\therefore \overleftrightarrow{AC}$  is a tangent

$$\therefore \overline{MC} \perp \overleftrightarrow{AC} \quad , \quad \therefore C \in \overline{MN}$$

$$\therefore \overline{NC} \perp \overleftrightarrow{AC} \quad ,$$

In circle N,  $\overline{AB}$  is a chord

$$, \quad \overline{NC} \perp \overleftrightarrow{AC}$$

In circle N,  $\therefore \overline{AB}$  is a chord

$$, \quad \therefore \overline{MN} \perp \overline{AB} \quad ,$$

$\therefore C$  is a mid point of  $\overline{AB}$

$\triangle \triangle AMC, BMC$

$$\left\{ \begin{array}{l} \therefore \overline{MC} \text{ is common side} \\ , CA = CB \\ , m(\angle MCA) = m(\angle MCB) \end{array} \right.$$

$$\therefore \triangle AMC \equiv \triangle BMC$$

$$\therefore MA = MB$$

(5)  $\therefore M, N$  are two intersecting circles,  $\overline{AB}$  is the common chord.

$$\therefore \overline{MN} \perp \overline{AB} \quad , \quad \therefore m(\angle AFN) = 90^\circ$$

$\therefore E$  is a mid point of  $\overline{CD}$

$$\therefore \overline{FM} \perp \overline{CD} \quad , \quad m(\angle CEF) = 90^\circ$$

$$\therefore m(\angle AFN) = m(\angle CEF) = 90^\circ$$

$$\therefore \overline{CD} \parallel \overline{AB}$$

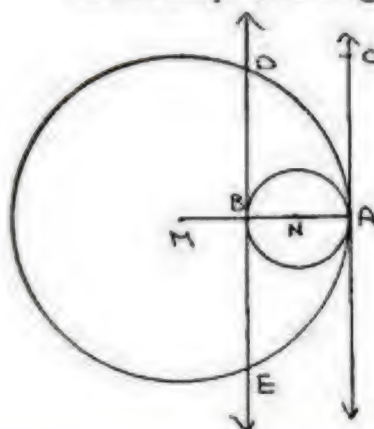
(6)  $\therefore \overleftrightarrow{AC}$  is the common tangent

$$\therefore \overline{MN} \perp \overleftrightarrow{AC}$$

$\therefore B \in \overline{MN}$ ,  $\overleftrightarrow{ED}$  tangent (N)

$$\therefore \overline{MN} \perp \overline{DE}$$

corresponding angles





$$\therefore \overrightarrow{CA} \parallel \overrightarrow{DE}$$

$$\therefore \overline{MA} \perp \overline{DE} \Rightarrow \overline{DE} \text{ is a chord in circle M}$$

$$\therefore B \text{ is the mid point of } \overline{DE}$$

$$\therefore BD = BE$$

(7) construction  $\therefore$  Draw  $\overline{AX}$  and  $\overline{DY}$

$$\therefore \overline{AB}$$
 is a tangent to circle M at A

$$\therefore \overline{MA} \perp \overline{AB}, \therefore \overline{AC} \parallel \overline{FD}$$

$$\therefore m(\angle AXE) = 90^\circ$$

$$\therefore \overline{DE}$$
 is a tangent to circle N at D

$$\therefore \overline{ND} \perp \overline{DE}, \therefore \overline{AC} \parallel \overline{FD}$$

$$\therefore m(\angle DYB) = 90^\circ$$

$$\therefore AXDY \text{ is a rectangle} \therefore AX = DY$$

$$\therefore M \text{ and } N \text{ are two congruent circles}$$

$$\therefore MA = ND \therefore MX = NY$$

$$\therefore \overline{MX} \perp \overline{EF}, \overline{NY} \perp \overline{BC}$$

$$\therefore EF = BC \quad (1) \quad (1^{st})$$

$$\therefore Ay = XD \quad (2)$$

$$\therefore \overline{MX} \perp \overline{EF}$$

$$\therefore x \text{ is a mid point of } \overline{EF}$$

$$\text{similarly } y \text{ is the mid point of } \overline{BC}$$

$$\therefore EF = BC \therefore BY = XE \quad (3)$$

$$\text{Subtracting (3) from (2)}$$

$$\therefore AB = DE \quad (2^{nd})$$



(8) construction: Draw  $\overline{ML} \perp \overleftrightarrow{EF}$

$\because$  x and y are mid points of  $\overline{AB}$  and  $\overline{AC}$  respectively.

$\therefore \overline{MX} \perp \overline{AB}$  and  $\overline{MY} \perp \overline{AC}$

$\because AB = CD$

$\therefore MX = MY$

$\Delta MXY$  is an isosceles  $\Delta$

$\therefore LX = LY$  (1)

$\because \overline{ML} \perp$  the chord  $\overline{EF}$

$\therefore EL = LF$  (2)

By subtracting (1) from (2)

We get  $XE = YF$

(9)  $\because$  MN are two intersecting circles.

$\therefore \overline{MN} \perp \overline{AB}$

$\because$  x is the mid point of  $\overline{AC}$

$\therefore \overline{NX} \perp \overline{AC}$

$\because AB = AC$

$\therefore NX = NY$

(10)  $\because$  E is the mid point of  $\overline{AB}$

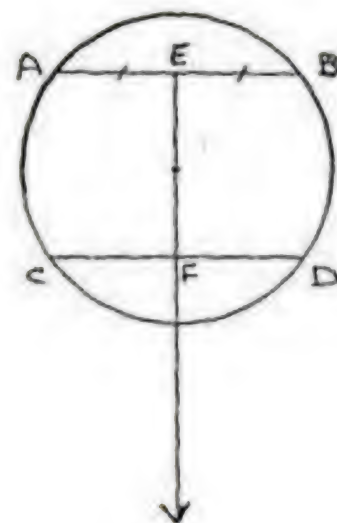
$\therefore \overline{ME} \perp \overline{AB}$

$\because \overline{AB} \parallel \overline{CD}$

$\therefore \overline{MF} \perp \overline{CD}$

$\therefore$  F is a mid point of  $\overline{CD}$

$\therefore FC = FD$





(11)  $\because E, D$  are the mid point of  $\overline{AB}$  and  $\overline{AC}$  respectively

$$\therefore \overline{MD} \perp \overline{AB}$$

$$\overline{ME} \perp \overline{AC}$$

In  $\triangle EMF$

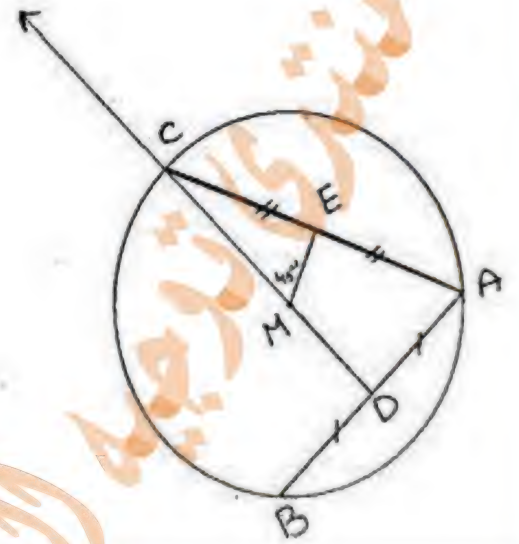
$$\because m(\angle MEF) = 90^\circ, EM = EF$$

$$\therefore m(\angle EMF) = \frac{180-90}{2} = 45^\circ$$

$$m(\angle EMD) = 180^\circ - 45^\circ = 135^\circ$$

In the quad ADME

$$m(\angle A) = 360^\circ - (90^\circ + 90^\circ + 135^\circ) = 45^\circ$$



(12) constructions: Draw  $\overline{AC}$  and  $\overline{BD}$

$$\because \overline{CD} \parallel \overline{AB}$$

$$\therefore m(\widehat{DB}) = m(\widehat{AC})$$

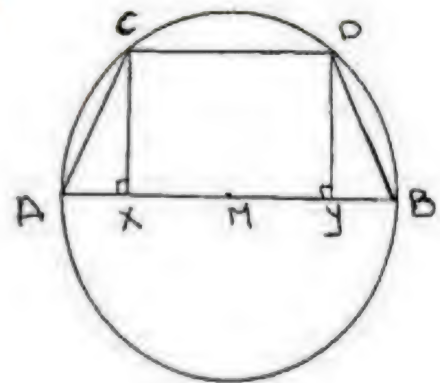
$$\therefore AC = BD$$

$\triangle AXC$  and  $\triangle BYD$

$$\left\{ \begin{array}{l} m(\angle DYB) = m(\angle CXA) = 90^\circ \\ CX = DY \\ AC = DB \end{array} \right.$$

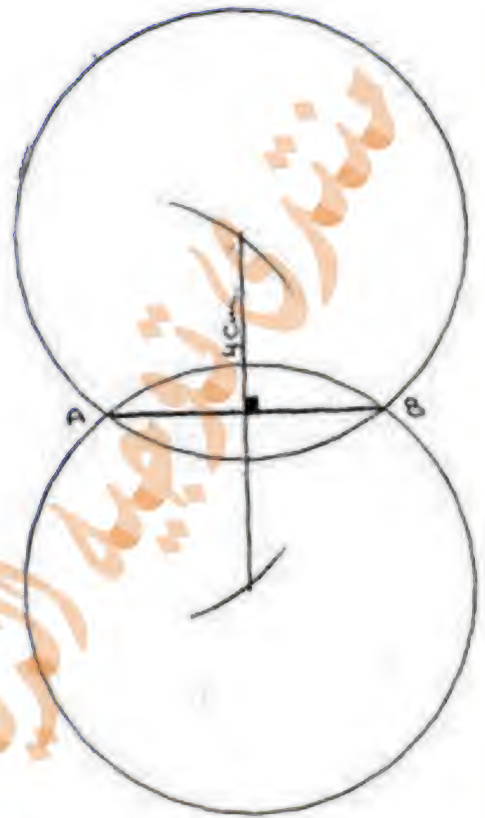
$$\therefore \triangle AXC \cong \triangle BYD$$

$$\therefore AX = YB$$

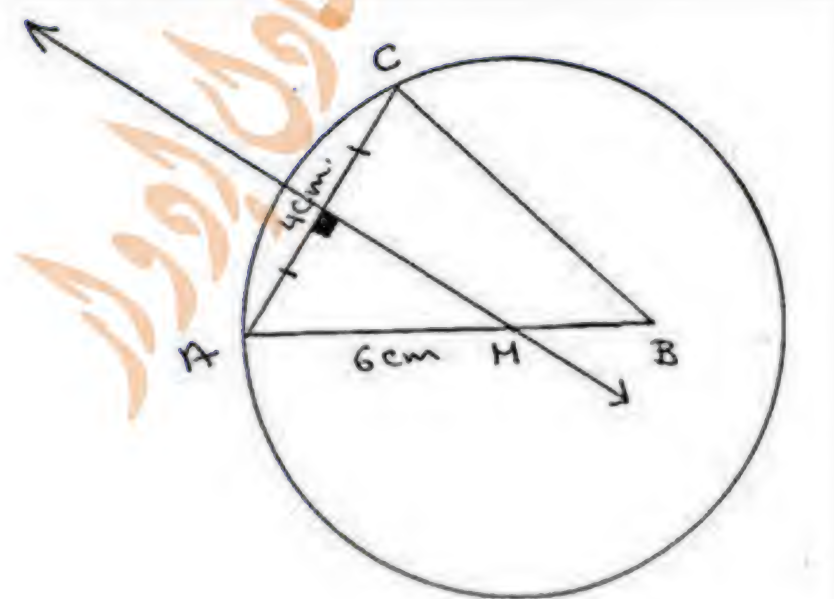




- (13) The distance is the length of the perpendicular from the centre of the circle to  $\overline{AB}$   
Distance = 4 cm.



(14)





(15)  $\because \overline{AB}$  is a diameter

$$\therefore m(\angle ACB) = 90^\circ$$

inscribed (semicircle)

$$\therefore m(\angle ADM) = m(\angle ACB) = 90^\circ$$

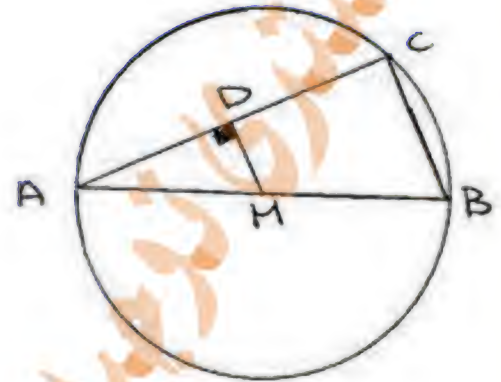
(corresponding)

$$\therefore \overline{DM} \parallel \overline{CB}$$

$\therefore \triangle ABC$  right angled at c

$$\therefore m(\angle A) = 30^\circ$$

$$\therefore CB = \frac{1}{2} AB$$



### General Exercise on the 2<sup>nd</sup> unit

#### First:

- (1) supplementary
- (2) equal in length
- (3) central angle subtended by the same arc.
- (4) (1)  $132^\circ$       (2)  $264^\circ$       (5) measure
- (6)      a)  $36^\circ$       b)  $72^\circ$       c)  $144^\circ$
- (7) acute angle      (8) equal
- (9) the inscribed angle subtended by this arc.

#### Second :Choose:

- (1) c      (2) c      (3) d      (4) d      (5) d      (6) d,a



## Questions Part (2)

### (1) Choose:

1- The angle of tangency included between .....

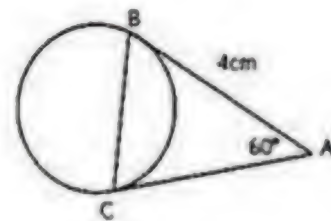
- a- two chords                      b- two tangents  
c- chord and tangent              d- chord and diameter

2- The number of tangents can be drawn from a point lies on a circle equals.

3- The number of common tangents can be drawn to two concentric circles equals .....

- a-zero              b-one              c-two              d-three

4-in the opposite figure  $\overline{AB}$  ,  $\overline{AC}$  are two tangents ,  $m(\angle A) = 60^\circ$  If  $AB = 4$  cm, then the length of  $\overline{CB}$  equals .....



- a-3cm              b-4cm              c-5cm              d-8cm

5- The number of common tangents can be drawn to two touching internally circles equals .....

- a- one              b-two              c- three              d- four



6- Using the following figures choose the correct answer

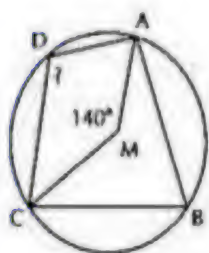


Figure (1)

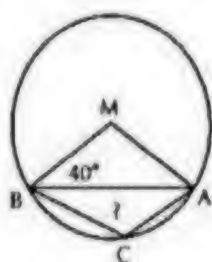


Figure (2)

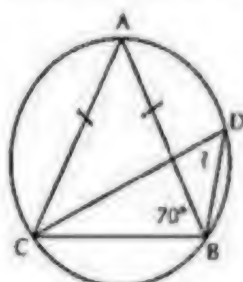


Figure (3)

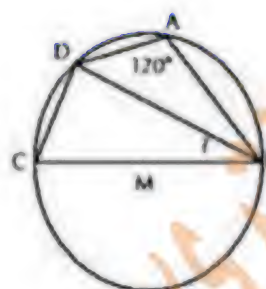


Figure (4)

In figure 1: A circle of center M ,  $m(\angle AMC) = 140^\circ$  , then  $m(\angle ADC) = \dots^\circ$

- a-  $40^\circ$       b-  $70^\circ$       c-  $110^\circ$       d-  $140^\circ$

In figure 2: if  $m(\angle ABM) = 40^\circ$  , then  $m(\angle ACB) = \dots^\circ$

- a-  $80^\circ$       b-  $100^\circ$       c-  $130^\circ$       d-  $140^\circ$

In figure 3: if  $m(\angle ABC) = 70^\circ$  , then  $m(\angle BDC) = \dots^\circ$

- a-  $20^\circ$       b-  $40^\circ$       c-  $60^\circ$       d-  $90^\circ$

In figure 4: if  $m(\angle BAD) = 120^\circ$  , then  $m(\angle CBD) = \dots^\circ$

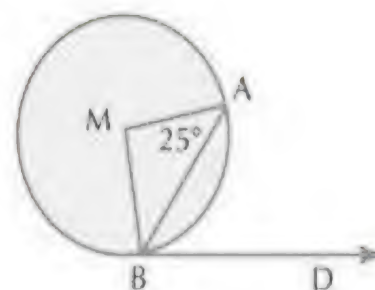
- a-  $15^\circ$       b-  $30^\circ$       c-  $45^\circ$       d-  $60^\circ$

7- In the opposite figure:

if  $\overrightarrow{BD}$  is a tangent to the circle M,

$m(\angle BAM) = 25^\circ$  , then  $m(\angle ABD) = \dots^\circ$

- a-  $25^\circ$       b-  $50^\circ$       c-  $65^\circ$       d-  $130^\circ$

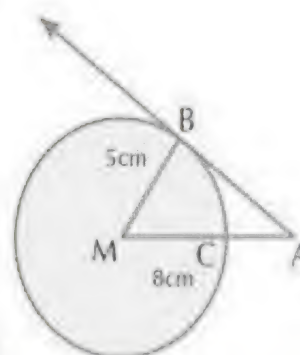


8- In the opposite figure:

$\overrightarrow{BA}$  is a tangent to the circle M ,

if  $MB = 5\text{cm}$  ,  $AC = 8\text{cm}$  , then  $AB = \dots\text{cm}$

- a- 5cm      b- 10cm      c- 12cm      d- 13cm





9- It is possible to draw a circle passing through the vertices of a .....

- a- trapezium    b- rhombus    c- parallelogram    d- rectangle

10-in the opposite figure

If  $m(\widehat{XZ}) = 70^\circ$  ,  $m(\widehat{YN}) = 30^\circ$  , then  $m(\angle E) = \dots\dots\dots^\circ$

- a-  $20^\circ$     b-  $40^\circ$     c-  $50^\circ$     d-  $100^\circ$



**(2) Answer the following questions**

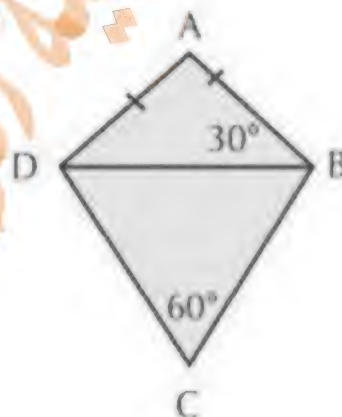
(1) (a) Prove that the two opposite angles in a cyclic quad are supplementary.

(b) In the opposite figure

ABCD is a quadrilateral in which  $AB=AD$ ,

$m(\angle ABD) = 30^\circ$  and  $m(\angle C) = 60^\circ$  prove that

ABCD is a cyclic quad.



(2) ABCD is a cyclic quadrilateral in which  $\overline{AB} \parallel \overline{CD}$  , if E is the mid point of  $\widehat{AB}$  . prove that :  $EC= ED$  .

(3) In the opposite figure

$\overline{MC} \cap \overline{AB} = \{c\}$  ,  $\overline{MC}$  intersects the circle at D.

$m(\angle MAB) = 20^\circ$  , Find

- 1-  $m(\widehat{AD})$     2-  $m(\angle DEB)$  .



(4) ABC is an acute angled triangle drawn inside a circle. draw  $\overline{AD} \perp \overline{BC}$  to cut  $\overline{BC}$  at D and cuts the circle at E circle , then draw  $\overline{CN} \perp \overline{AB}$  to cut  $\overline{AB}$  at N . prove that:

1- ANDC is a cyclic quad.

2-  $m(\angle BND) = m(\angle BED)$



(5) ABC is an equilateral triangle drawn inside a circle, D is a point on the arc AB, E is a point on  $\overline{DC}$  such that  $AD = DE$ . Prove that:

1- ADE is an equilateral triangle.

2-  $\overline{DB} \parallel \overline{AE}$

3-  $m(\angle DCB) = m(\angle EAC)$

4-  $DB = EC$

(6) In the opposite figure :

ABC is a triangle in which  $AB = AC$ .  $\overline{BC}$  is a chord in the circle M, if  $\overline{AB}$ ,  $\overline{AC}$  cut the circle at D, E prove that:

$\overline{BC} \parallel \overline{DE}$  and if  $m(\angle DCA) = 30^\circ$  and  $m(\angle A) = 50^\circ$ , find:

1-  $m(\angle BEC)$

2-  $m(\angle BMC)$

3-  $m(\angle CDE)$



(7) (a) prove that the angles subtended by the same arcs in the circle are equal in measure .

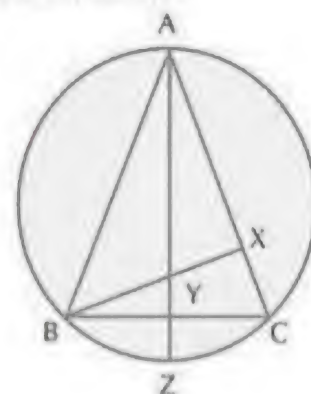
(b) In the opposite figure ABC is a triangle

in a circle,  $\overline{BX} \perp \overline{AC}$ ,  $\overline{AY} \perp \overline{BC}$

Cuts it at Y and cuts the circle at Z, prove that:

1- ABYX is a cyclic quad.

2-  $\overline{BC}$  bisects  $\angle XBZ$



(8) In the opposite figure

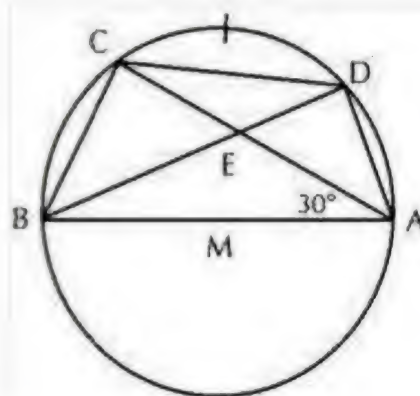
$\overline{AB}$  is a diameter of a circle M, CE the circle,

$m(\angle CAB) = 30^\circ$ , D is the mid-point

of the arc  $\widehat{AC}$  and  $\overline{DB} \cap \overline{AC} = \{E\}$ .

1- find  $m(\angle BDC)$ ,  $m(\angle ABD)$

2- prove that  $\triangle ABE$  is an isosceles triangle.

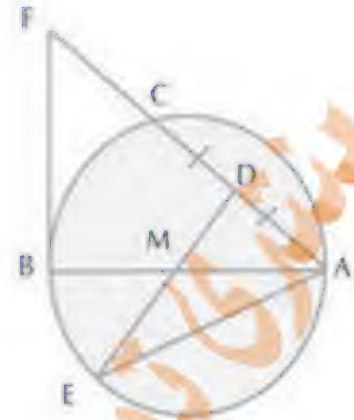




(9) In the opposite figure

$\overline{AB}$  is a diameter of a circle M, D is the mid-point of the arc  $\overline{AC}$ . Draw  $\overline{DM}$  to cut the circle at E,  $\overline{BF}$  is a tangent to the circle to cut  $\overline{AC}$  at F. prove that :

- 1- MBFD is a cyclic quad.      2-  $\overline{DE} \parallel \overline{BC}$ .

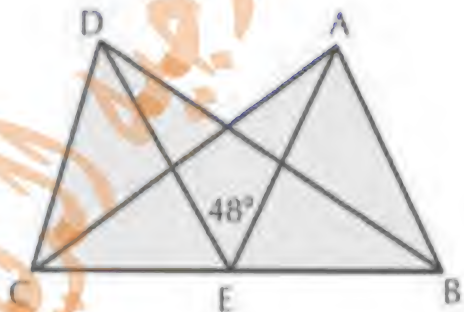


(10) In the opposite figure

$$m(\angle BAC) = m(\angle BDC) = 90^\circ$$

E is the mid-point of  $\overline{BC}$  and  $m(\angle AED) = 48^\circ$

- 1- find  $m(\angle ABD)$   
2- prove that: (a)  $m(\angle ABD) = m(\angle ACD)$   
(b)  $m(\angle AEC) = 2m(\angle ABC)$



(11) ABCD is a quadrilateral drawn in a circle, draw  $\overline{EF} \parallel \overline{CB}$  to cut  $\overline{CD}$  at E cuts  $\overline{AB}$  at F,  $\overline{DF} \cap \overline{CB} = \{x\}$ . prove that:

- 1- AFED is a cyclic quad.  
2-  $m(\angle BXF) = m(\angle EAD)$

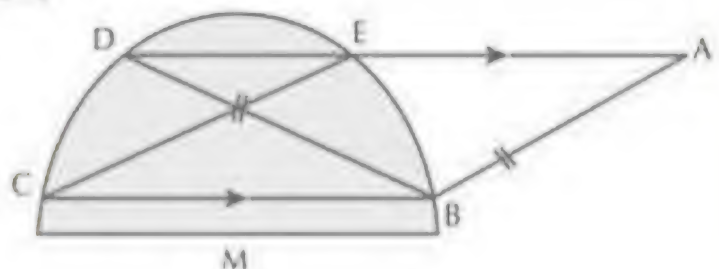
(12) A is a point outside a circle, draw  $\overline{AB}$  to cut the circle at B, C respectively, then draw  $\overline{AD}$  to cut the circle at D, E respectively, if  $AC = AE$  prove that:

- 1-  $\overline{BD} \parallel \overline{CE}$       2-  $m(\widehat{BC}) = m(\widehat{ED})$

(13) In the opposite figure

A semicircle of center M,  $\overline{AD} \parallel \overline{BC}$ ,

Prove that ABCE is a parallelogram.





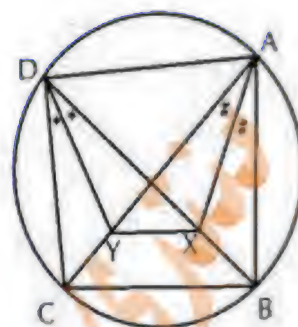
(14) In the opposite figure

ABCD is a quadrilateral in a circle M,  $\overline{AX}$  bisects  $\angle BAC$ ,

$\overline{DY}$  bisects  $\angle BDC$ , prove that:

1- AXYD is a cyclic quad.

2-  $\overline{XY} \parallel \overline{BC}$

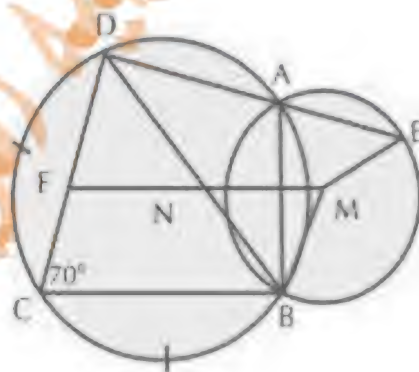


(15) In the opposite figure

$m(\angle C) = 70^\circ$ , the length of  $\widehat{CD}$  = the length of  $\widehat{BC}$

$\overline{MN} \cap \overline{CD} = \{F\}$  and  $\overline{DA} \cap \text{the circle} = \{E\}$ . find

$m(\angle BDC)$ ,  $m(\angle BAD)$  and  $m(\angle BME)$



(16)  $\overline{AB}$  is a diameter of a circle M,  $D \in \overline{AB}$ ,  $D \notin \overline{AB}$ . draw the tangent  $\overline{DC}$  at C, draw  $\overline{CE}$ , if  $E \in \overline{CB}$  such that  $DE = DC$  prove that:

1- ACDE is a cyclic quad .

2-  $\overline{AE}$  is the diameter of the circumcircle of the figure ACDE.

3-  $\overline{DE}$  is the tangent of the circumcircle of the triangle ABE.



## Model Answers Part (2)

(1) Choose:

- |                  |        |      |      |       |
|------------------|--------|------|------|-------|
| 1) c             | 2) one | 3) a | 4) b | 5) a  |
| 6) c , c , b , b | 7) c   | 8) c | 9) d | 10) a |

Answer the Following P.170

(1) (a) Given:

ABCD is cyclic quad .

R.T.P.  $m(\angle A) + m(\angle C) = 180^\circ$

$m(\angle B) + m(\angle D) = 180^\circ$

proof :  $\because m(\angle A) = \frac{1}{2} m(\widehat{BCD})$  and  $m(\angle C) = \frac{1}{2} m(\widehat{BAD})$

$\because m(\angle A) + m(\angle C) = \frac{1}{2} [m(\widehat{BCD}) + m(\widehat{BAD})] = \frac{1}{2} m. \text{ of the circle}$

$= \frac{1}{2} \times 360^\circ = 180^\circ$  similarly  $m(\angle B) + m(\angle D) = 180^\circ$

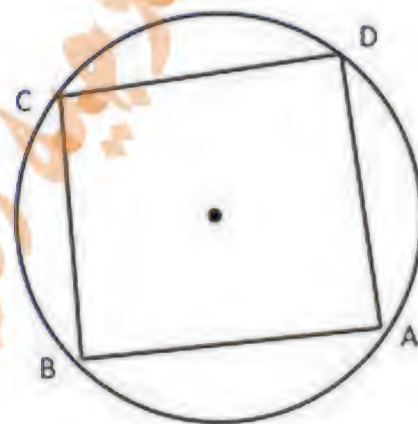
(b)  $\because \Delta ABD$  is an isosceles  $\Delta$

$\because m(\angle ABD) = 30^\circ$

$\therefore m(\angle A) = 180^\circ - (30^\circ + 30^\circ) = 120^\circ$

$\because m(\angle A) + m(\angle C) = 120^\circ + 60^\circ = 180^\circ$

$\therefore ABCD$  is acyclic quad.





$$(2) \overline{AB} \parallel \overline{CD} \therefore m(\widehat{BC}) = m(\widehat{AD}) \quad [1]$$

$\therefore E$  is the midpoint of  $(AB)$

$$\therefore m(\widehat{BE}) = m(\widehat{AE}) \quad [2]$$

by adding [1] and [2]

$$\therefore m(\widehat{CE}) = m(\widehat{DE})$$

$$\therefore EC = ED$$

$$(3) \because \overline{MC} \perp \overline{AB}, \therefore m(\angle A) = 20^\circ$$

$$\therefore m(\angle AMC) = 180 - (90 + 20) = 70^\circ$$

$(\angle AMD)$  is a central angle subtended by arc  $(\widehat{AD})$

$$\therefore m(\widehat{AD}) = 70^\circ, \because \overline{MC} \perp \overline{AB} \therefore CA = CB$$

$\Delta \Delta MCA$  and  $MCB$

$$\because MA = MB = r$$

$$\because AC = BC$$

$$\because m(\angle MCA) = m(\angle MCB) = 90^\circ$$

$$\therefore \Delta MCA \equiv \Delta MCB$$

$$\therefore m(\angle AMC) = m(\angle BMC) = 70^\circ \quad \therefore m(\angle BED) = \frac{1}{2} m(\angle BMD)$$

$$= 70 \div 2 = 35^\circ \text{ (inscribed and central angles subtended by the same arc } (\widehat{BD}))$$

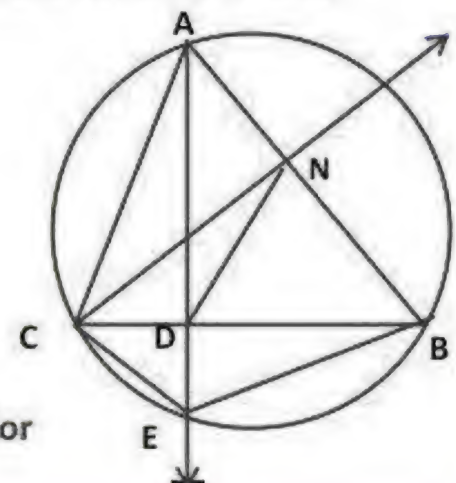
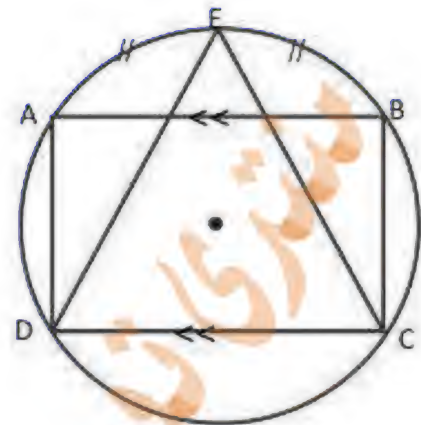
$$(4) \because \overline{AD} \perp \overline{BC}, \therefore \overline{CN} \perp \overline{AB}$$

$$\therefore m(\angle ANC) = m(\angle ADC) = 90^\circ$$

Subtended by the chord  $\overline{AC}$

and on one side of it  $\therefore ANDC$  is cyclic quad.

$$\therefore m(\angle BND) = m(\angle ACD) \text{ exterior} = \text{opposite interior}$$





$\therefore (\angle BEA)$  and  $(\angle BCA)$  two inscribed angles subtended by same are  $(\widehat{AB})$

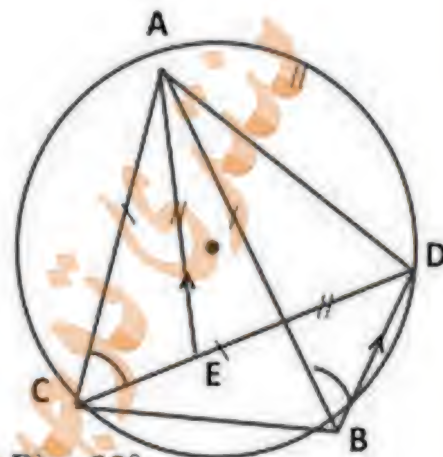
$$\therefore m(\angle BED) = m(\angle BND)$$

(5)  $\therefore \Delta ABC$  is an equilateral  $\Delta$

$$\therefore m(\angle B) = 60^\circ$$

$\therefore \angle B, \angle D$  are two inscribed angles

subtended by the same are  $(\widehat{AC}) \therefore m(\angle D) = m(\angle B) = 60^\circ$



$$\therefore AD = DE \quad \therefore m(\angle DAE) = m(\angle AED) = \frac{180-60}{2} = 60^\circ$$

$\therefore \Delta ADE$  is an equilateral  $\Delta$

[1]

$\angle CDB$  and  $\angle CAB$  are two inscribed angles subtended by the same are  $\widehat{CB}$

$$\therefore m(\angle CDB) = m(\angle CAB) = 60^\circ \quad \therefore m(\angle AED) = 60^\circ \text{ (equilateral } \Delta)$$

$$\therefore m(\angle AED) = m(\angle EDB) = 60^\circ \text{ alternate angles}$$

$$\therefore \overline{AE} \parallel \overline{DB}$$

[2]

$$\therefore \Delta ADE \text{ is an equilateral } \Delta \quad \therefore m(\angle AED) = 60^\circ \text{ (exterior angle)}$$

$$\therefore m(\angle AED) = m(\angle EAC) + m(\angle ECA) \quad \Rightarrow \angle EAC + \angle ECA = 60^\circ \rightarrow 1$$

$$\therefore \angle DCB + \angle ECA = 60^\circ \rightarrow 2$$

$$\text{From 1, 2} \quad \therefore m(\angle DCB) = m(\angle EAC) \rightarrow 3$$

In  $\Delta ADB, AEC \quad \therefore \angle BCD, \angle BAD$  are 2 inscribed angles subtended by same are  $(\widehat{BD})$

$$\therefore m(\angle BCD) = m(\angle BAD) \rightarrow 4$$

From 3,4

$$\therefore m(\angle BAD) = m(\angle EAC) \quad \& \quad AB = AC \quad \& \quad AE = AD$$

$$\therefore \Delta ADB \equiv \Delta AEC$$



$$\therefore DB = EC$$

(6) In  $\triangle ABC$ ,  $\because AB = AC \therefore \triangle ABC$  is an isosceles  $\triangle$ .

$$\therefore m(\angle ABC) = m(\angle ACB)$$

$$\therefore m(\widehat{DEC}) = m(\widehat{EDB}) \rightarrow 1$$

Subtract  $m(\widehat{ED})$  from 1  $\therefore m(\widehat{EC}) = m(\widehat{DB}) \therefore \overline{BC} \parallel \overline{DE}$

$$\because m(\angle A) = 50^\circ, AB = AC$$

$$\therefore m(\angle ACB) = \frac{180^\circ - 50^\circ}{2} = 65^\circ \quad \therefore m(\angle DCA) = 30^\circ$$

$$\therefore m(\angle DCB) = 35^\circ \quad \therefore m(\angle DBC) = 65^\circ$$

$$\therefore \text{In } \triangle DBC \quad \therefore m(\angle BDC) = 180^\circ - (35^\circ + 65^\circ) = 80^\circ$$

$\therefore \angle BDC, \angle BEC$  subtended by same arc  $\widehat{BC}$

$\therefore m(\angle BEC) = 80^\circ$ ,  $\therefore m(\angle BHC), m(\angle BDC)$  are central, inscribed angles subtended by same arc  $\widehat{BC}$ .

$$\therefore m(\angle BMC) = 2m(\angle BDC) = 2 \times 80^\circ$$

$$\therefore m(\angle BMC) = 160^\circ$$

$$\because \overline{AB} \text{ is a straight line, } \therefore m(\angle BDC) = 80^\circ$$

$$\therefore m(\angle ADC) = 100^\circ, \therefore \overline{ED} \parallel \overline{BC}$$

$$\therefore m(\angle ADE) = 65^\circ$$

$$\therefore m(\angle CDE) = 100^\circ - 65^\circ = 35^\circ$$



Given:  $\angle C, \angle D$  are inscribed angles subtended by same arc  $\widehat{AB}$

R.t.P.:  $m(\angle C) = m(\angle D)$

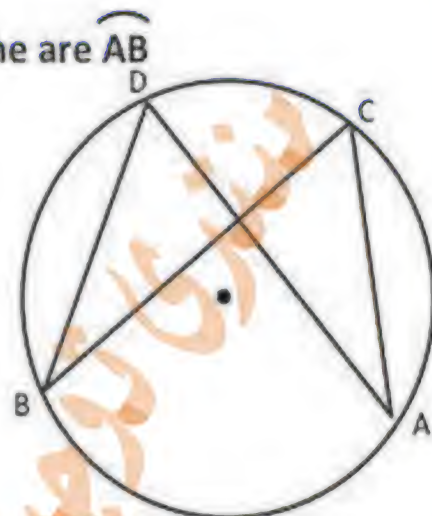
$$\therefore m(\angle C) = \frac{1}{2} m \widehat{AB}, m(\angle D) = \frac{1}{2} m \widehat{AB}$$

$$\therefore m(\angle C) = m(\angle D)$$

$$(ii) \text{ In } \triangle ABC \therefore \overline{BX} \perp \overline{AC} \therefore m(\angle AXB) = 90^\circ$$

$$\therefore \overline{AY} \perp \overline{BC} \therefore m(\angle AYB) = 90^\circ$$

$$\therefore m(\angle AXB) = m(\angle AYB)$$



& drawn on one side of its sides as a base, on one side

$\therefore$  AXBY is a cyclic quadrilateral

$$\therefore m(\angle XAY) = m(\angle YBX) \text{ \& }$$

$\therefore (\angle XAY), (\angle XBY)$  are subtended by same arc  $\widehat{XC}$

$$\therefore m(\angle XBY) = m(\angle YBX)$$

$\therefore \overline{BC}$  bisects  $\angle XBY$

$$(8) \therefore \overline{AB} \text{ is a diameter } \therefore m(\angle ACB) = 90^\circ$$

$$\therefore m(\angle BAC) = 30^\circ \text{ \& } m(\angle BDA) = 90^\circ$$

$$\therefore m(\angle ABC) = 180^\circ - (90^\circ + 30^\circ) = 60^\circ$$

$$\therefore D \text{ is midpoint of arc } \widehat{AC} \therefore \widehat{AD} = \widehat{DC}$$

$$\therefore m(\angle ABD) = m(\angle DBC) = \frac{60^\circ}{2} = 30^\circ$$

$$\therefore m(\angle ABD) = 30^\circ$$

$\therefore$  ABCD is a cyclic quadrilateral

$$\therefore m(\angle ABC) + m(\angle ADC) = 180^\circ$$

$$\therefore 60^\circ + m(\angle BDC) + m(\angle BDA) = 180^\circ$$



$$\therefore m(\angle BDC) = 180^\circ - (60^\circ + 90^\circ) = 30^\circ$$

$$\therefore m(\angle BDC) = 30^\circ$$

In  $\triangle BCE, ADE$

$$\therefore m(\angle BCA) = m(\angle BDA) = 90^\circ$$

$$\& m(\angle CBD) = m(\angle CAD) \quad (\text{subtended by same Arc } \widehat{CD})$$

$$\& m(\angle BEC) = m(\angle AED) \text{ (V. O. A)}$$

$$\therefore \triangle BCE \equiv \triangle ADE \quad \therefore BE = EA$$

(9)  $\therefore \overline{BF}$  is a tangent to circle M ( with a diameter )

$$\therefore m(\angle ABF) = 90^\circ \quad [1]$$

$$\& \therefore D \text{ is midpoint of } \overline{AC} \quad \therefore \overline{MD} \perp \overline{AC}$$

$$\therefore m(\angle MDF) = 90^\circ \quad \therefore m(\angle ABF) + m(\angle MDF) = 180^\circ$$

$\therefore$  MBFD is a cyclic quadrilateral

$\therefore \overline{AB}$  is a diameter .

$\therefore$  M is midpoint of  $\overline{AB}$  & D is midpoint of  $\overline{AC}$

$$\therefore \overline{DE} \parallel \overline{BC}$$

(10) construction : Draw  $\overline{AD}$

Proof:  $\therefore m(\angle BAC) = m(\angle BDC) = 90^\circ$

$\therefore$  figure ABCD is a cyclic quadrilateral and  $\overline{BC}$  is a diameter in the circumcircle of it

$\therefore$  E is midpoint of  $\overline{BC}$

$\therefore$  E is Centre of circle which passes through points A, B, C and D.

$$\therefore m(\angle ABD) = \frac{1}{2} m(\angle AED) = 24^\circ$$

inscribed angle & central angle of same arc (  $\widehat{AD}$  )



$\therefore$  ABCD is a cyclic quadrilateral

$\therefore m(\angle ABD) = m(\angle ACD)$  (drawn on  $\overline{AD}$  & on one side of it )

&  $\therefore m(\angle BAD) = m(\angle BDC) = 90^\circ$

$\therefore \overline{BC}$  is a diameter &  $\therefore E$  is the midpoint of  $\overline{BC}$

$\therefore E$  is the center of circum circle of ABCD

$\therefore \angle ABC$  &  $\angle AEC$  are inscribed & central angle subtended by arc  $\widehat{AC}$

$\therefore m(\angle AEC) = 2m(\angle ABC)$

(11)  $\therefore \overline{EF} \parallel \overline{BC}$

$\therefore m(\angle FED) = m(\angle BCD) \rightarrow \boxed{1}$  (corresponding angles )

&  $\therefore$  ABCD is acyclic quad .

$\therefore m(\angle FAD) + m(\angle BCD) = 180^\circ \rightarrow \boxed{2}$

From 1 & 2

$\therefore m(\angle FAD) + m(\angle FED) = 180^\circ$

$\therefore$  AFED is a cyclic quad .

$\therefore \overline{EF} \parallel \overline{BC}$

$\therefore m(\angle EFD) = m(\angle BCF)$  ( corresponding angles )

&  $m(\angle EFD) = m(\angle EAD)$  (drawn on  $\overline{ED}$  and on side of it ) &

( AFED is a cyclic quad )

$\therefore m(\angle BCF) = m(\angle EAD)$

(12) In  $\triangle ABC$   $\therefore AE = AC$

$\therefore \triangle ABC$  is an isosceles  $\triangle$

$\therefore m(\angle ACE) = m(\angle AEC) \rightarrow \boxed{1}$

$\therefore$  CBDE is acyclic quad .



$$\left. \begin{array}{l} \therefore m(\angle CED) + m(\angle CBD) = 180^\circ \\ \& m(\angle ECB) + m(\angle BDE) = 180^\circ \end{array} \right\} (2)$$

From 1 & 2

$$\therefore m(\angle DBC) + m(\angle BCE) = 180^\circ \& m(\angle BDE) + m(\angle DEC) = 180^\circ$$

$$\therefore \overline{BD} \parallel \overline{CE} \quad \therefore m(\widehat{BC}) = m(\widehat{ED})$$

$$(13) \text{ In } \triangle ABD \quad \therefore AB = BD$$

$$\therefore m(\angle A) = m(\angle BDA)$$

$$\therefore m(\angle BCE) = m(\angle BDE) \quad (2 \text{ inscribed angles subtended by } \widehat{BE})$$

$$\therefore m(\angle A) = m(\angle BCE) \rightarrow [1]$$

$$\therefore \overline{AD} \parallel \overline{BC} \therefore \angle A \text{ supplements } \angle ABC, \angle BCE \text{ supplements } \angle AEC$$

$$\therefore m(\angle ABC) = m(\angle AEC) \rightarrow [2]$$

From 1 & 2

$$\therefore ABCE \text{ is a parallelogram.}$$

$$(14) \therefore ABCD \text{ is a cyclic quad.}$$

$$\therefore m(\angle BAC) = m(\angle BDC) \text{ (drawn on } \overline{BC} \text{ & on one side of it)}$$

$$\therefore \frac{1}{2} m(\angle BAC) = \frac{1}{2} m(\angle BDC)$$

$$\therefore m(\angle XAY) = m(\angle XDY) \text{ but they are draw on } \overline{XY} \text{ & on one side of it}$$

$$\therefore AX YD \text{ is a cyclic quad} \quad \therefore ABCD \text{ is a cyclic quad}$$

$$\therefore m(\angle CBD) = m(\angle CAD) \rightarrow [1] \text{ (drawn on } \overline{CD} \text{ & on one side of it)}$$

$$\therefore AX YD \text{ is a cyclic quad}$$

$$\therefore m(\angle YXD) = m(\angle YAD) \rightarrow [2] \text{ from 1 \& 2}$$

$$\therefore m(\angle CBD) = m(\angle YXD) \quad \text{similarly} \quad \therefore m(\angle BCA) = m(\angle XYA)$$

$$\therefore \overline{XY} \parallel \overline{BC}$$









- 1 Two distance circles M and N with radii lengths 6 cm and 8 cm respectively , then MN ..... 14 cm. « Cairo 2019 »  
 (a)  $>$  (b)  $\geq$  (c)  $<$  (d)  $=$
- 2 The measure of inscribed angle is ..... the measure of the central angle subtended by the same Arc .  
 (a) Half (b) Twice (c) Quarter (d) Third « Cairo 2019 »
- 3 In the cyclic quad , if :  $m(\angle A) = \frac{1}{2} m(\angle C)$  , then :  $m(\angle A) =$  .....  
 (a)  $20^\circ$  (b)  $30^\circ$  (c)  $60^\circ$  (d)  $120^\circ$  « Cairo 2019 »
- 4 The measure of inscribed angle in a semicircle = ..... « Giza , S.sinaï 2019 »  
 (a)  $45^\circ$  (b)  $90^\circ$  (c)  $120^\circ$  (d)  $180^\circ$
- 5 Two circles M and N touching internally , their radii lengths 3 cm and 5 cm respectively , then MN = ..... cm. « Giza 2019 »  
 (a) 3 (b) 5 (c) 2 (d) 8
- 6 If the surface of circle M  $\cap$  the surface of the circle N = { A } and the radius length of one of them equals 3 cm. , and MN = 8 cm , then the radius length of the other circle = ..... cm. « Alex 2019 »  
 (a) 5 (b) 6 (c) 11 (d) 16
- 7 A circle can be drawn passing through the vertices of a ..... « Alex 2019 , sharkia 2019 »  
 (a) Rhombus (b) Parallelogram (c) Trapezium (d) Rectangle
- 8 A circle with diameter length equals 10 cm. , the straight line L is distant from its center by 5 cm. , then the straight line L is ..... « sharkia 2019 »  
 (a) a tangent (b) a secant  
 (c) Outside the circle (d) a diameter of the circle
- 9 The number of common tangents of two touching circles externally equals ..... « sharkia 2019 »  
 (a) zero (b) 1 (c) 2 (d) 3
- 10 If M and N are two touching circles externally , the lengths of their radii are 2 cm. , and 4 cm. Respectively , then the area of the circle with diameter  $\overline{MN}$  equals .....  $\text{cm}^2$  « sharkia 2019 »  
 (a)  $36\pi$  (b)  $9\pi$  (c)  $16\pi$  (d)  $4\pi$
- 11 M and N are two circles , whose radii lengths are 6 cm. , and 8 cm. and MN = 14 cm. Then the two circles are ..... « Dakahlia 2019 »  
 (a) Distant (b) Intersecting  
 (c) One inside the other (d) Touching externally





- 12 A circle with greatest chord with length = 12 cm. , then the circumference of the circle = .....  
« Dakahlia 2019 »  
 (a)  $12\pi$  (b)  $24\pi$  (c)  $6\pi$  (d)  $10\pi$
- 13 The inscribed angle drawn in a semicircle is ..... « Dakahlia , r.sea 2019 »  
 (a) an acute (b) a straight (c) an obtuse (d) a right
- 14 A chord is of length 8 cm. in a circle of diameter length 10 cm.  
 , then the chord is at ..... from the center of the circle. « Dakahlia 2019 »  
 (a) 2 cm. (b) 3 cm. (c) 4 cm. (d) 6 cm.
- 15 The number of common tangents of two touching circles internally is ..... « Dakahlia 2019 »  
 (a) zero (b) 1 (c) 2 (d) 3
- 16 In the cyclic quad , if :  $m(\angle A) + 3 m(\angle C) = 280^\circ$  , then :  $m(\angle C) =$  .....  
 (a)  $50^\circ$  (b)  $150^\circ$  (c)  $130^\circ$  (d)  $100^\circ$
- 17 the measure of the central angle drawn in  $\frac{1}{3}$  circle equals ..... « ismailia 2019 »  
 (a)  $240^\circ$  (b)  $120^\circ$  (c)  $60^\circ$  (d)  $30^\circ$
- 18 Which of the following figures is cyclic quadrilateral ..... « ismailia 2019 »  
 (a) the Rhombus (b) the Parallelogram (c) the Trapezium (d) the Rectangle
- 19 If  $AB = 8$  cm. then the radius length of the smallest circle can be drawn passing through  
 the two points A and B equals ..... cm. « ismailia 2019 »  
 (a) 1 (b) 2 (c) 3 (d) 4
- 20 If M and N are two intersecting circles whose radii length are 5 cm. and 2 cm.  
 , Then :  $MN \in$  ..... « kalyoubia 2019 »  
 (a)  $]3, 7[$  (b)  $[3, 7]$  (c)  $[3, 7[$  (d)  $]3, 7]$
- 21 the measure of the central angle which is opposite to an arc of length  $\frac{1}{3}\pi r$  ..... « kalyoubia 2019 »  
 (a)  $30^\circ$  (b)  $60^\circ$  (c)  $120^\circ$  (d)  $240^\circ$
- 22 The axis of symmetry of a circle is ..... « Monofia 2019 »  
 (a) The diameter (b) The chord  
 (c) The tangent (d) The straight line passing through the center
- 23 ABCD is a cyclic quad in which ,  $m(\angle A) = 2 m(\angle C)$  , then :  $m(\angle A) =$  ..... « Dakahlia , monofia 2019 »  
 (a)  $30^\circ$  (b)  $60^\circ$  (c)  $90^\circ$  (d)  $120^\circ$
- 24 The longest chord in the circle is called a ..... « p.said 2019 »  
 (a) diameter (b) Tangent (c) chord (d) radius





- 25 If the two circles M and N touching externally and the radius length of one of them equals 3 cm. , and  $MN = 8$  cm , then the radius length of the other circle = ..... cm. « Suez 2019 »  
 (a) 5 (b) 6 (c) 11 (d) 16
- 26 If the straight line L is a tangent to the circle M of diameter length equals 10 cm. , then the distance between L and the center of the circle equals ..... cm. « p.said 2019 »  
 (a) 3 (b) 4 (c) 5 (d) 10
- 27 The ratio between The measure of the inscribed angle and the measure of the central angle subtended by the same Arc is ..... « matrouh 2019 »  
 (a) 1 : 2 (b) 2 : 1 (c) 3 : 2 (d) 2 : 3
- 28 A chord with length 8 cm. in a circle with circumference  $10\pi$  cm. , then it is distant from its center by ..... cm. « matrouh 2019 »  
 (a) 2 (b) 3 (c) 4 (d) 5
- 29 The angle of tangency is included between ..... « r.sea 2019 »  
 (a) Two chords (b) Two tangents  
 (c) a chord and a tangent (d) a chord and a diameter
- 30 The number of symmetry axis of the semicircle is ..... « r.sea 2019 »  
 (a) 0 (b) 1 (c) 3 (d) an infinite number
- 31 The number of symmetry axis of the circle is ..... « S.sinai , sohaq 2019 »  
 (a) 0 (b) 1 (c) 3 (d) an infinite number
- 32 the diameter length of the circle whose center is the origin point and passes through the point  $(3, -4)$  equals ..... length unit.  
 (a) 2.5 (b) 5 (c) 10 (d) 20
- 33 If the surface of circle  $M \cap$  the surface of the circle  $N = \{A\}$ , then M and N are ..... « N.sinai 2019 »  
 (a) distant (b) Concentric  
 (c) Touching externally (d) Intersecting
- 34 ABCD is a cyclic quadrilateral , then :  $m(\angle A) + m(\angle C) - 80^\circ =$  ..... « Aswan 2019 »  
 (a)  $60^\circ$  (b)  $80^\circ$  (c)  $100^\circ$  (d)  $120^\circ$
- 35 The length of the arc subtending a central angle of measure  $60^\circ$  in a circle whose circumference Is 24 cm. equals ..... cm. « Luxor 2019 »  
 (a) 4 (b) 8 (c) 12 (d) 16
- 36 If A , B two points in the plane ,  $AB = 7$  cm. then the diameter length of the smallest circle passing through the two points A and B equals ..... cm. « Qena 2019 »  
 (a) 3 (b) 3.5 (c) 7 (d) 14





- 37 The diameter is a ..... passing through the center of the circle. « Assiut 2019 »  
 (a) ray (b) Straight line (c) tangent (d) chord
- 38 If the circumference of a circle is  $20\pi$  cm. , then its area = .....  $\text{cm}^2$   
 (a) 10 (b) 20 (c)  $100\pi$  (d)  $400\pi$
- 39 The symmetry axis of the common chord  $\overline{AB}$  of the two intersecting circles M , N is ..... « B.suef 2019 »  
 (a)  $\overrightarrow{MA}$  (b)  $\overrightarrow{MN}$  (c)  $\overrightarrow{MB}$  (d)  $\overrightarrow{AB}$
- 40 If M is circle of diameter length 8 cm. , the straight line L is far from the center M of the circle 4 cm. , then the straight line L is ..... « Fayoum 2019 »  
 (a) a secant to the circle in two points. (b) Outside the circle.  
 (c) A tangent to the circle. (d) an axis of symmetry of the circle.
- 41 the center of the circle that passing through the vertices of the triangle is the intersection point of ..... « Fayoum 2019 »  
 (a) The bisectors of its interior angles. (b) The bisectors of its exterior angles.  
 (c) Its altitudes. (d) The axis of its sides.
- 42 If M is circle of diameter length 8 cm. , the straight line L is far from the center M of the circle 4 cm. , then the straight line L is ..... « Fayoum 2019 »
- 43 If the straight line L is a tangent to the circle M of diameter length equals 8 cm. , then L is at a distance of ..... cm. from the centre. « kalyoubia 2018 »  
 (a) 3 (b) 4 (c) 5 (d) 10
- 44 If M is circle , its diameter length = 14 cm. ,  $MA = (2x + 3)$  cm. where A is a point on the circle. , then :  $x =$  ..... « Sharkia 2015 »  
 (a) 1 (b) 2 (c) 3 (d) 5
- 45 A circle of circumference  $6\pi$  cm. , and the straight line L is distance from its centre by 3 cm. , then the straight line L is ..... « monofia 2015 »  
 (a) a diameter of the circle. (b) a secant.  
 (c) A tangent to the circle. (d) Outside the circle.
- 46 If M is circle , its diameter length =  $(2x + 5)$  cm. , and the straight line L is distance  $(x + 2)$  cm. from its centre circle , then the straight line L is ..... « P.said 2017 »  
 (a) a secant to the circle in two points. (b) Outside the circle.  
 (c) A tangent to the circle. (d) an axis of symmetry of the circle.





- 47 Two circles M and N with radii lengths 4 cm. and 7 cm. respectively , are touching  
 , Then :  $MN \in$  .....  
 (a)  $] 3, 11[$  (b)  $[ 3, 11]$  (c)  $\mathbb{R} - [ 3, 7]$  (d)  $\{ 3, 11 \}$
- 48 If the radii lengths of the two circles M and N are 6 cm. and 3 cm.  $MN = 2$  cm. circle.  
 , then the two circles M and N are ..... « *Dakahlia 2018* »  
 (a) Distant (b) Intersecting  
 (c) One inside the other (d) Touching externally
- 49 The number of circles passing through three collinear points is ..... « *Giza 2016 , Souhag 2018* »  
 (a) zero (b) one (c) three (d) an infinite number
- 50 The number of circles passing through three collinear points is ..... « *menia 2017* »  
 (a) zero (b) one (c) two (d) three
- 51 The type of the inscribed angle which is opposite to an arc greater than the semicircle  
 is ..... « *N.vally 2018* »  
 (a) acute (b) obtuse (c) straight (d) right
- 52 The centre of the circles passing through the two points A and B lies on ..... « *menia 2017* »  
 (a) the axis of symmetry of  $\overline{AB}$  (b)  $\overline{AB}$   
 (c) The perpendicular to  $\overline{AB}$  (d) the midpoint of  $\overline{AB}$
- 53 The length of the arc which represents  $\frac{1}{4}$  the circumference of the circle = ..... cm. « *menia 2017* »  
 (a)  $2\pi r$  (b)  $\pi r$  (c)  $\frac{1}{2}\pi r$  (d)  $4\pi r$
- 54 Its impossible to draw a circle passing through the vertices of a ..... « *B.suef 2017* »  
 (a) rectangle (b) triangle (c) square (d) rhombus
- 55 The inscribed angle which is subtended by minor arc in a circle is ..... « *Qena 2016* »  
 (a) acute (b) obtuse (c) straight (d) right
- 56 The number of tangents can be drawn from a point lies on a circle is ..... « *Beheira 2017* »  
 (a) 1 (b) 2 (c) 3 (d) Infinite number
- 57 The number of common tangents of two intersecting circles is .....  
 (a) 1 (b) 2 (c) 3 (d) 4
- 58 The number of common tangents of two distant circles is .....  
 (a) 1 (b) 2 (c) 3 (d) 4
- 59 The ratio between The measure of the inscribed angle and the measure of the angle of the tangency  
 subtended by the same Arc is ..... «  
 (a) 1 : 2 (b) 2 : 1 (c) 1 : 1 (d) 1 : 3





- 60 If :  $\overleftrightarrow{AB} \cap \text{the circle } M = \{A, B\}$ , then :  $\overleftrightarrow{AB} \cap \text{the surface of the circle } M =$  .....
- (a)  $\{A, B\}$  (b)  $\overline{AB}$  (c)  $\overrightarrow{AB}$  (d)  $\overleftrightarrow{AB}$
- 61 If  $\overline{MA}$  and  $\overline{MB}$  are two perpendicular radii in the circle  $M$  and the area of the triangle  $MAB = 8 \text{ cm}^2$ , then the radius length of the circle = ..... cm.
- (a) 2 (b) 4 (c) 8 (d) 16
- 62 A circle of radius length = 2 cm. , then its circumference = ..... cm. « Aswan 2016 »
- (a)  $4\pi$  (b)  $5\pi$  (c)  $6\pi$  (d)  $7\pi$
- 63 the two opposite angles in the cyclic quadrilateral are ..... « Alex 2017 »
- (a) equal (b) Supplementary (c) Complementary (d) Alternate
- 64 If : the circle  $M \cap \text{the circle } N = \{A, B\}$ , then the two circles are ..... « Ismailia 2018 »
- (a) Distant (b) Intersecting  
(c) Concentric (d) Touching
- 65 ABCD is a cyclic quadrilateral , in which :  $m(\angle A) = 75^\circ$ , then :  $m(\angle C) =$  ..... « R.sea 2016 »
- (a)  $75^\circ$  (b)  $125^\circ$  (c)  $150^\circ$  (d)  $105^\circ$
- 66 Which of the following points doesn't belong to the circle whose centre is the origin and its radius length = 7 cm ? « Giza 2016 »
- (a) (0, 7) (b) (0, -7) (c) (7, 0) (d) (7, 7)
- 67 ABCDEF is a regular hexagon drawn inside the circle  $M$ , then :  $m(\widehat{BC}) =$  .....
- (a)  $30^\circ$  (b)  $60^\circ$  (c)  $90^\circ$  (d)  $120^\circ$
- 68 A circle with diameter length =  $(2x)$  cm. , and the straight line  $L$  is distance  $(x + 1)$  cm. from its centre circle , then the straight line  $L$  will be ..... « Dakahlia 2018 »
- (a) secant. (b) Outside  
(c) tangent. (d) axis of symmetry.
- 69 ABC is an equilateral triangle drawn inscribed in circle  $M$ , then :  $m(\widehat{AB}) =$  ..... « Fayoum 2018 »
- (a)  $30^\circ$  (b)  $60^\circ$  (c)  $90^\circ$  (d)  $120^\circ$
- 70 the measure of the in inscribed angle which is drawn in  $\frac{1}{3}$  a circle equals ..... « Dakahlia 2018 »
- (a)  $240^\circ$  (b)  $120^\circ$  (c)  $60^\circ$  (d)  $30^\circ$
- 71  $\overline{AB}$  and  $\overline{DC}$  are two intersected chord at the point  $X$  in the circle  $M$ , and  $m(\widehat{AC}) + m(\widehat{BD}) = 130^\circ$ . Then  $m(\angle AXC) =$  .....
- (a)  $260^\circ$  (b)  $130^\circ$  (c)  $65^\circ$  (d)  $60^\circ$



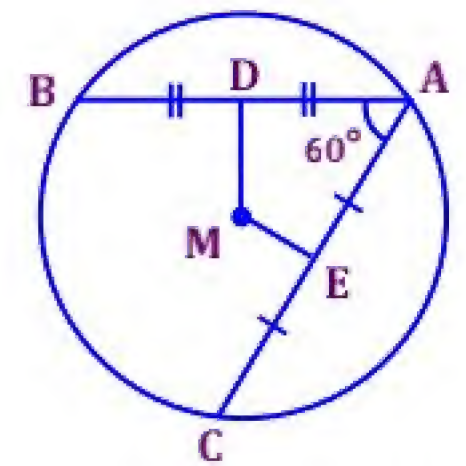


In each of the following figures, choose the correct answer

1 in the opposite figure :

If D and E is the midpoints of  $\overline{AB}$  and  $\overline{AC}$  , and  $m(\angle BAC) = 60^\circ$   
Then :  $m(\angle DME) = \dots\dots\dots$

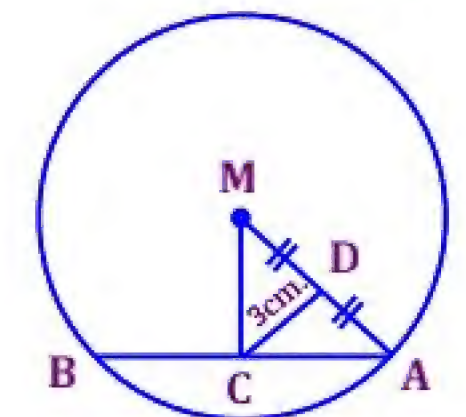
- (a)  $60^\circ$  (b)  $120^\circ$   
(c)  $90^\circ$  (d)  $30^\circ$



2 in the opposite figure : « Gharbia 2019 »

$\overline{AB}$  is a chord in circle M ,  $\overline{MC} \perp \overline{AB}$  , D is a midpoint of  $\overline{MA}$  ,  $CD = 3$  cm.  
 , Then the surface area of the circle =  $\dots\dots\dots \pi \text{ cm}^2$

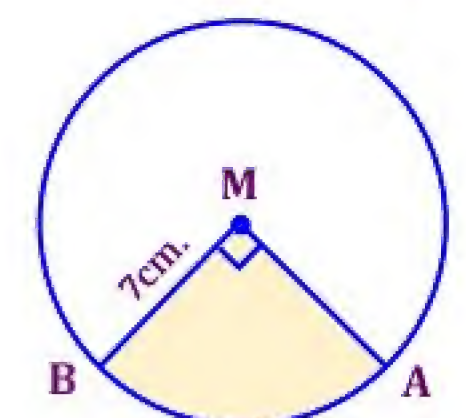
- (a) 3 (b) 6  
(c) 9 (d) 36



3 in the opposite figure : « Gharbia 2019 »

If  $\overline{MA}$  and  $\overline{MB}$  are two radii perpendicular in the circle M which its radius  
Length = 7 cm. then the perimeter of shaded shape =  $\dots\dots\dots \text{cm}$ . ( $\pi = \frac{22}{7}$ )

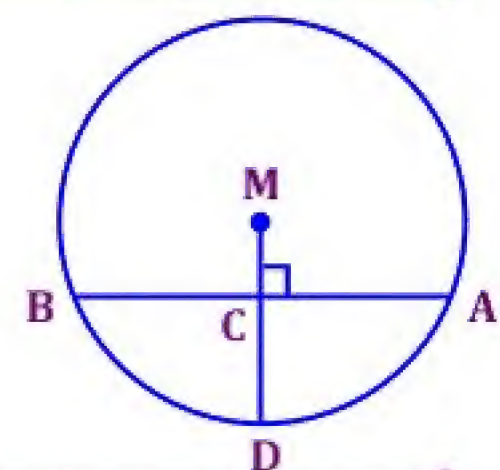
- (a) 14 (b) 11  
(c) 38.5 (d) 25



4 in the opposite figure :

Circle M with radius length = 13 cm. and  $AB = 24$  cm. , then :  $CD = \dots\dots\dots \text{cm}$ .

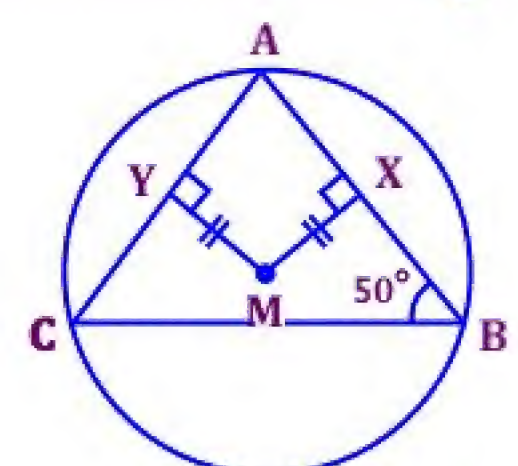
- (a) 6.5 (b) 8  
(c) 10 (d) 12



5 in the opposite figure :

$MX = MY$  and  $m(\angle B) = 50^\circ$  , then :  $m(\angle A) = \dots\dots\dots$

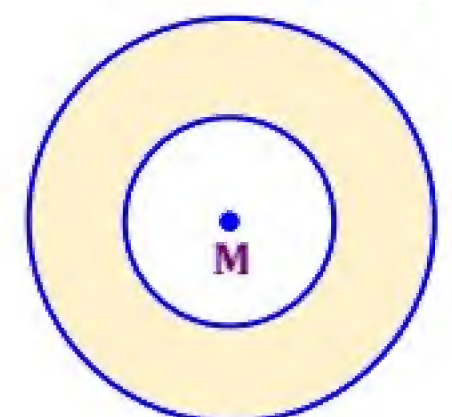
- (a)  $50^\circ$  (b)  $60^\circ$   
(c)  $70^\circ$  (d)  $80^\circ$



6 in the opposite figure :

Two concentric circle with centre M , their radii 7 cm. and 14 cm.  
Then the area of shaded shape =  $\dots\dots\dots \text{cm}^2$  «  $\pi = \frac{22}{7}$  »

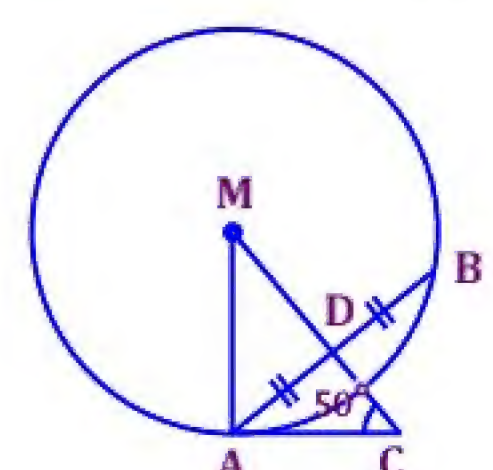
- (a) 315 (b) 412  
(c) 462 (d) 530



7 in the opposite figure :

$\overline{AC}$  is a tangent to the circle M at A , D is the midpoint of  $\overline{AB}$  and  $m(\angle C) = 50^\circ$   
 , then :  $m(\angle A) = \dots\dots\dots$

- (a)  $40^\circ$  (b)  $45^\circ$   
(c)  $50^\circ$  (d)  $90^\circ$

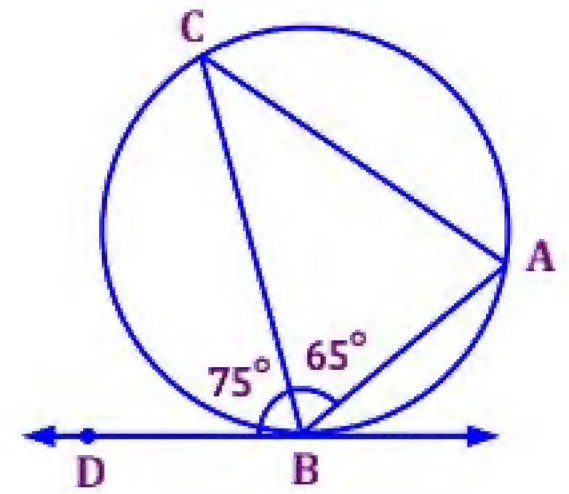




8 in the opposite figure :

$\overleftrightarrow{BD}$  is a tangent to the circle M at B ,  $m(\angle ABC) = 65^\circ$  and  $m(\angle DBC) = 75^\circ$   
 , Then :  $m(\angle DBC) = \dots\dots\dots$

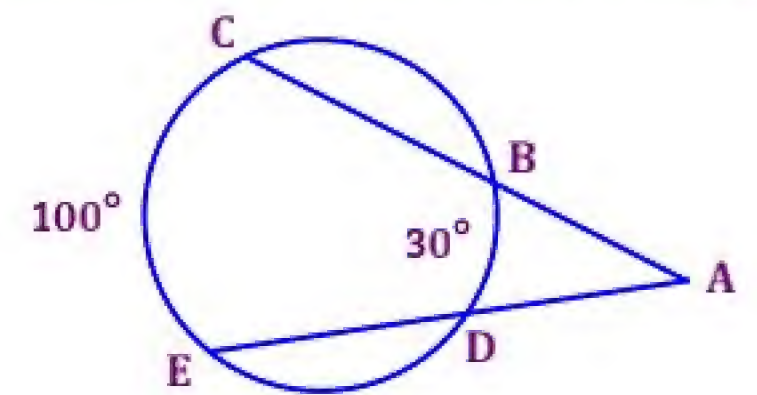
- (a)  $20^\circ$  (b)  $40^\circ$   
 (c)  $50^\circ$  (d)  $80^\circ$



9 in the opposite figure :

If :  $m(\widehat{CE}) = 100^\circ$  and  $m(\widehat{BD}) = 30^\circ$  , Then :  $m(\angle A) = \dots\dots\dots$

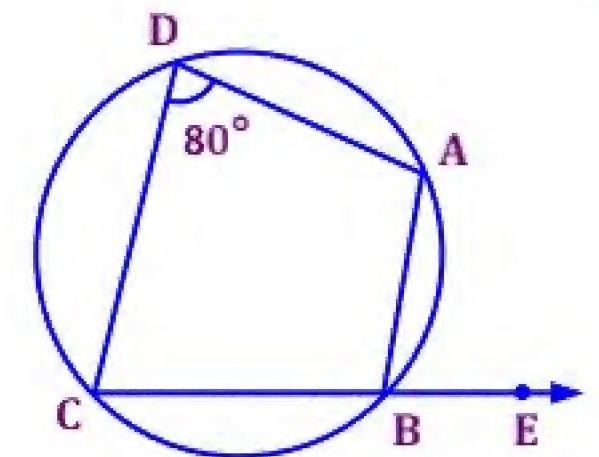
- (a)  $70^\circ$  (b)  $65^\circ$   
 (c)  $60^\circ$  (d)  $35^\circ$



10 in the opposite figure :

If :  $m(\angle ADC) = 80^\circ$  , Then :  $m(\angle ABE) = \dots\dots\dots$

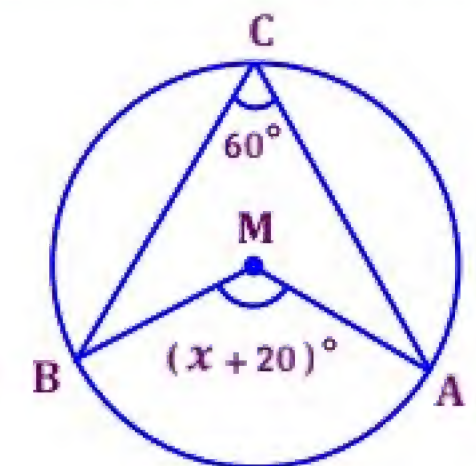
- (a)  $10^\circ$  (b)  $80^\circ$   
 (c)  $60^\circ$  (d)  $100^\circ$



11 in the opposite figure :

If :  $m(\angle ACB) = 60^\circ$  ,  $(\angle AMB) = (x + 20)^\circ$  , Then :  $x = \dots\dots\dots$

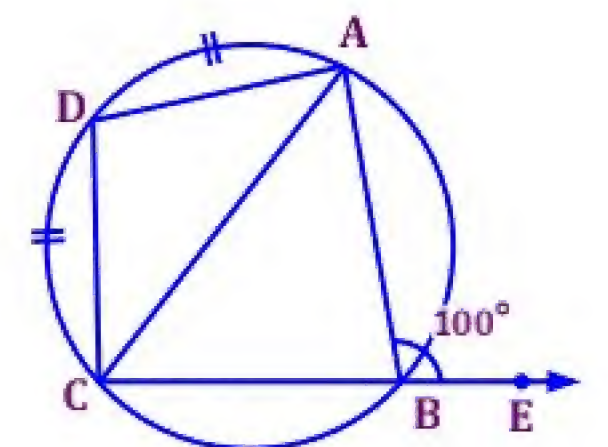
- (a)  $30^\circ$  (b)  $40^\circ$   
 (c)  $80^\circ$  (d)  $100^\circ$



12 in the opposite figure :

If :  $m(\angle ABE) = 100^\circ$  ,  $m(\widehat{AD}) = m(\widehat{DC})$  , Then :  $m(\angle ACD) = \dots\dots\dots$

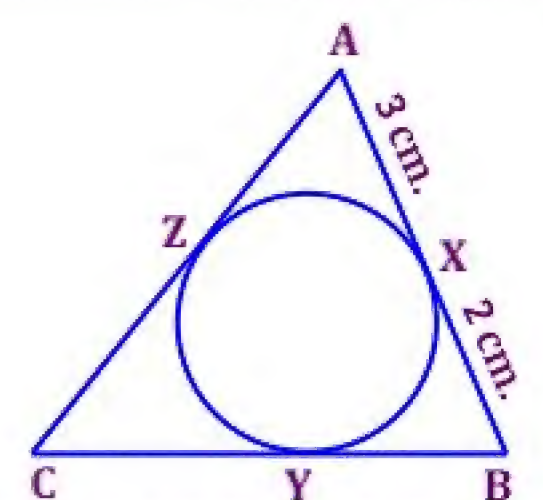
- (a)  $100^\circ$  (b)  $80^\circ$   
 (c)  $40^\circ$  (d)  $30^\circ$



13 in the opposite figure :

If :  $AX = 3$  cm. ,  $XB = 2$  cm. and ,  $AC = 8$  cm. Then :  $CB = \dots\dots\dots$  cm.

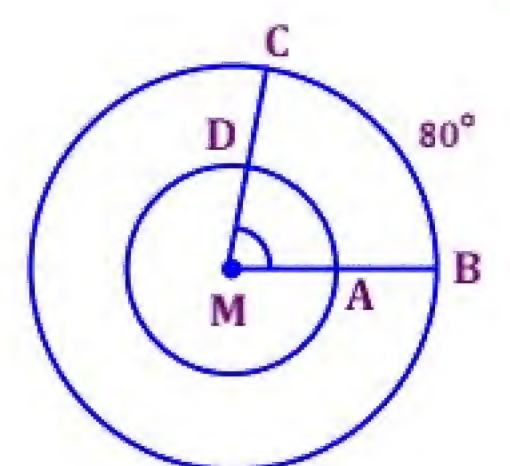
- (a) 5 (b) 7  
 (c) 10 (d) 13



14 in the opposite figure :

Two concentric circle with centre M ,  $m(\widehat{AC}) = 80^\circ$   
 , Then :  $m(\widehat{AD}) = \dots\dots\dots$

- (a)  $20^\circ$  (b)  $40^\circ$   
 (c)  $80^\circ$  (d)  $160^\circ$

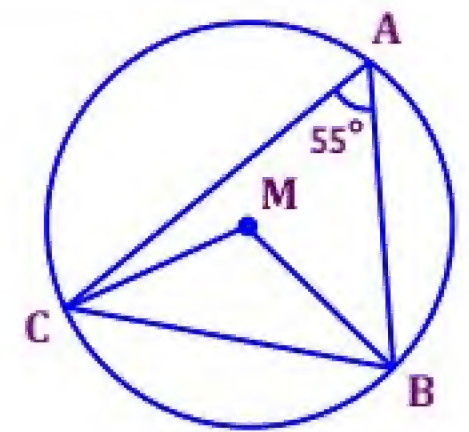




15 in the opposite figure :

If :  $m(\angle BAE) = 55^\circ$ , Then :  $m(\angle MBC) =$  .....

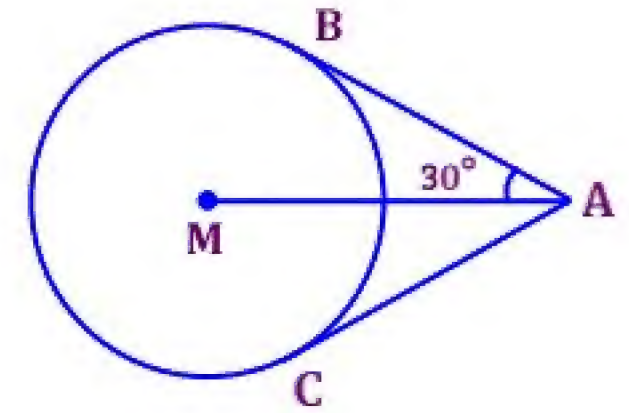
- (a)  $110^\circ$  (b)  $55^\circ$   
(c)  $35^\circ$  (d)  $25^\circ$



16 in the opposite figure :

$\overline{AB}$  and  $\overline{AC}$  Two tangents to the circle M from the point A ,  $m(\angle BAM) = 30^\circ$   
 , Then :  $AB =$  ..... cm.

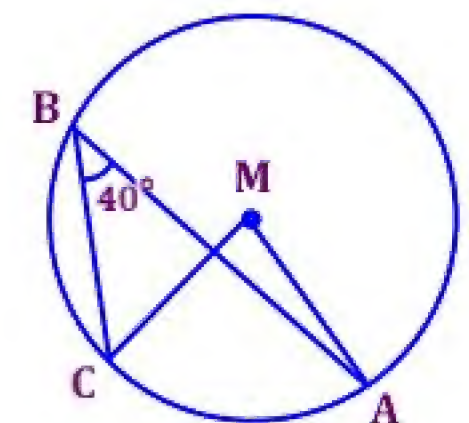
- (a) 8 (b)  $8\sqrt{3}$   
(c)  $4\sqrt{3}$  (d)  $2\sqrt{3}$



17 in the opposite figure :

If :  $m(\angle ABC) = 40^\circ$ , Then :  $m(\angle AMC) =$  .....

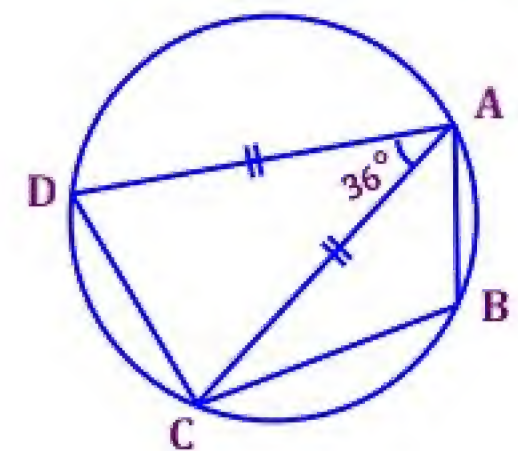
- (a)  $20^\circ$  (b)  $40^\circ$   
(c)  $80^\circ$  (d)  $140^\circ$



18 in the opposite figure :

If :  $m(\angle DAC) = 36^\circ$ , and  $AC = AD$  , Then :  $m(\angle B) =$  .....

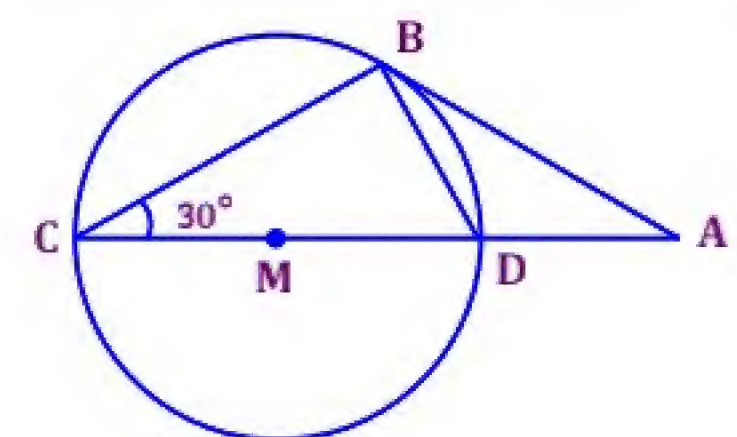
- (a)  $140^\circ$  (b)  $108^\circ$   
(c)  $70^\circ$  (d)  $40^\circ$



19 in the opposite figure :

$\overline{AB}$  is a diameter in circle M ,  $m(\angle BCD) = 30^\circ$ , Then :  $m(\angle ABC) =$  .....

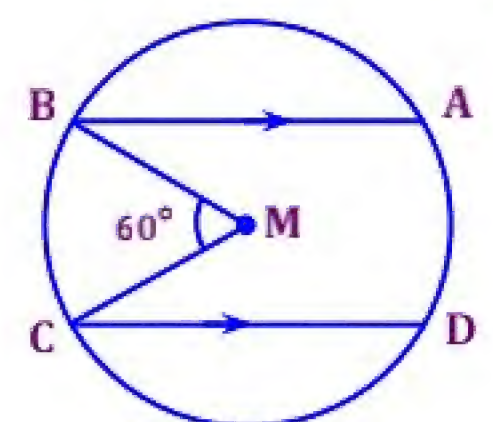
- (a)  $120^\circ$  (b)  $110^\circ$   
(c)  $90^\circ$  (d)  $30^\circ$



20 in the opposite figure :

$\overline{AB} \parallel \overline{CD}$  ,  $m(\angle BMC) = 60^\circ$ , Then :  $m(\widehat{AD}) =$  .....

- (a)  $30^\circ$  (b)  $60^\circ$   
(c)  $90^\circ$  (d)  $120^\circ$

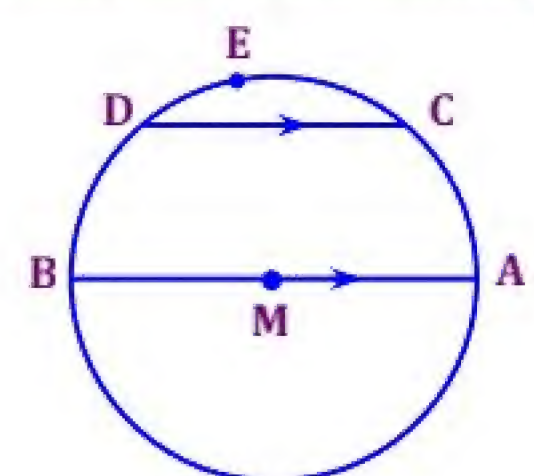


21 in the opposite figure :

$\overline{AB}$  is a diameter in circle M ,  $\overline{AB} \parallel \overline{CD}$  ,  $m(\widehat{DEC}) = 80^\circ$

, Then :  $m(\widehat{AC}) =$  .....

- (a)  $40^\circ$  (b)  $50^\circ$   
(c)  $80^\circ$  (d)  $100^\circ$

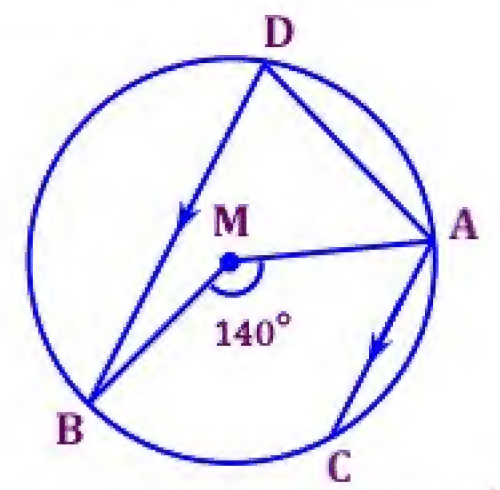




22 in the opposite figure :

$\overline{AC} \parallel \overline{BD}$ ,  $m(\angle AMB) = 140^\circ$ , Then :  $m(\angle DAC) =$  .....

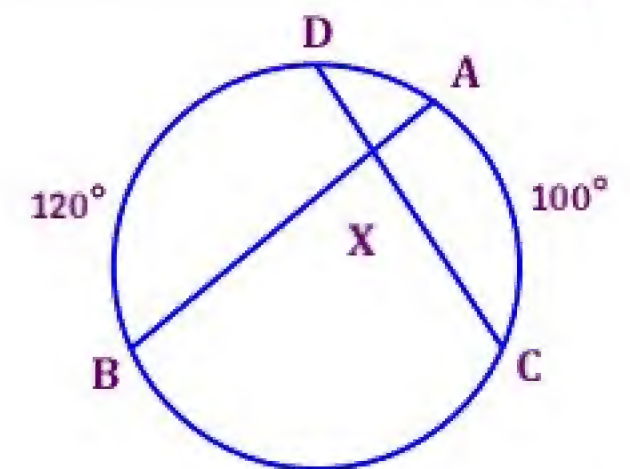
- (a)  $70^\circ$  (b)  $110^\circ$   
(c)  $140^\circ$  (d)  $220^\circ$



23 in the opposite figure :

$\overline{AB} \cap \overline{DC} = \{X\}$ ,  $m(\widehat{AC}) = 100^\circ$ ,  $m(\widehat{BD}) = 120^\circ$ , Then :  $m(\angle AXC) =$  .....

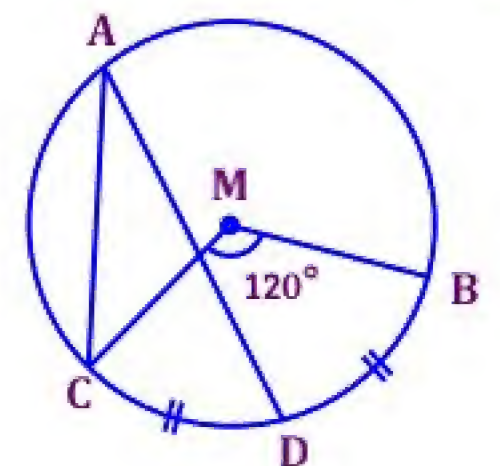
- (a)  $110^\circ$  (b)  $55^\circ$   
(c)  $70^\circ$  (d)  $140^\circ$



24 in the opposite figure :

D is a midpoint of the arc  $\widehat{CB}$ ,  $m(\angle CMB) = 120^\circ$ , Then :  $m(\angle A) =$  .....

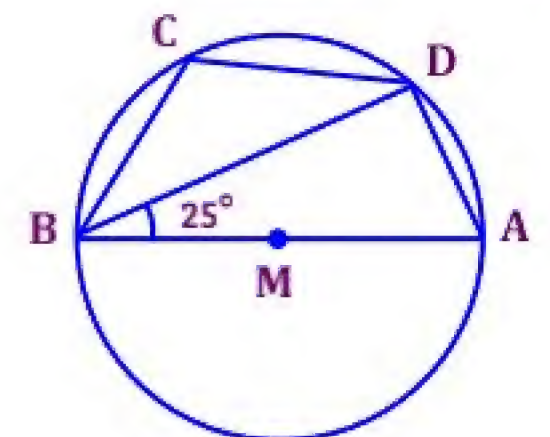
- (a)  $15^\circ$  (b)  $30^\circ$   
(c)  $15^\circ$  (d)  $60^\circ$



25 in the opposite figure :

$\overline{AB}$  is a diameter in circle M,  $m(\angle ABD) = 25^\circ$ , Then :  $m(\angle C) =$  .....

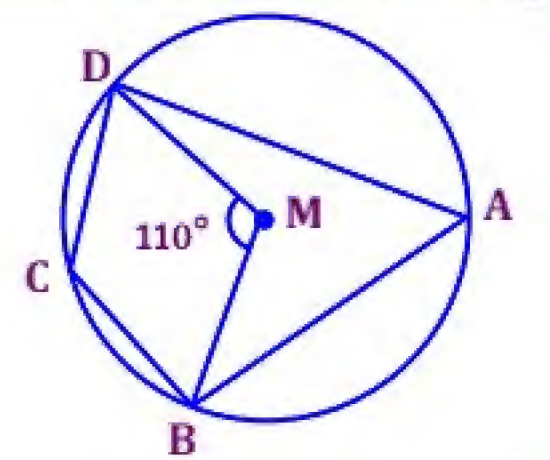
- (a)  $100^\circ$  (b)  $50^\circ$   
(c)  $115^\circ$  (d)  $125^\circ$



26 in the opposite figure :

If :  $m(\angle DMB) = 110^\circ$ , Then :  $m(\angle C) =$  .....

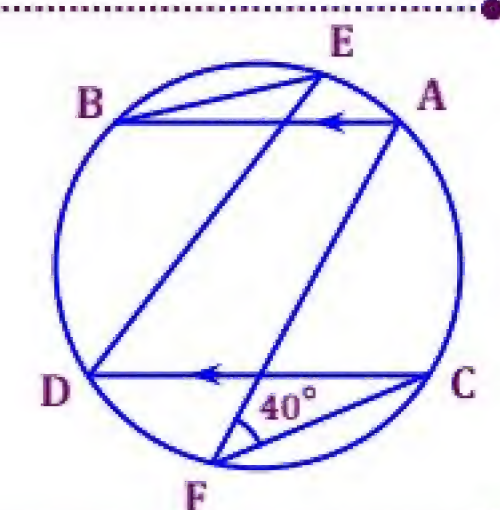
- (a)  $70^\circ$  (b)  $110^\circ$   
(c)  $55^\circ$  (d)  $125^\circ$



27 in the opposite figure :

$\overline{AB} \parallel \overline{CD}$ ,  $m(\angle AFC) = 40^\circ$ , Then :  $m(\angle BED) =$  .....

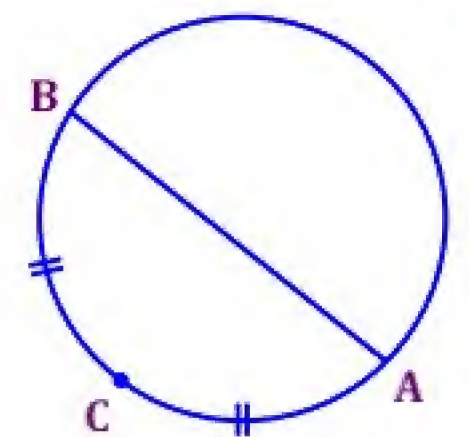
- (a)  $30^\circ$  (b)  $60^\circ$   
(c)  $90^\circ$  (d)  $120^\circ$



28 in the opposite figure :

If C is a midpoint of the arc  $\widehat{AB}$ , Then :  $AB$  .....  $2 AC$

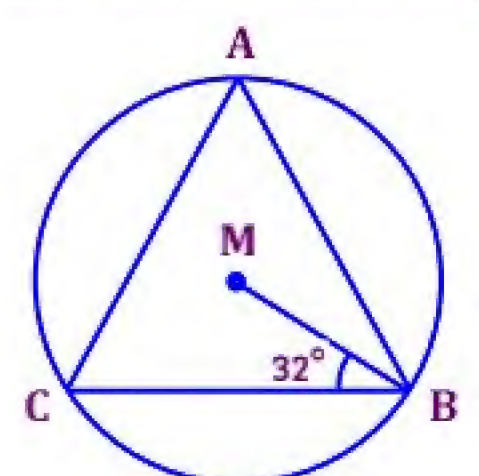
- (a)  $>$  (b)  $<$   
(c)  $=$  (d)  $\leq$



29 in the opposite figure :

If :  $m(\angle MBC) = 32^\circ$ , Then :  $m(\angle A) =$  .....

- (a)  $16^\circ$  (b)  $32^\circ$   
(c)  $116^\circ$  (d)  $64^\circ$

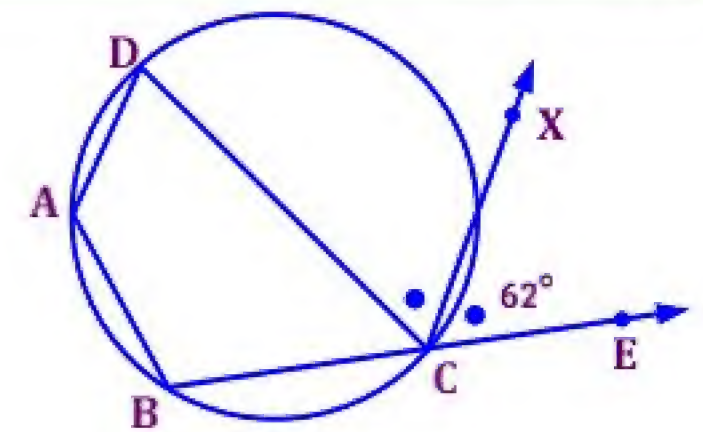




30 in the opposite figure :

$\overrightarrow{CX}$  bisects  $\angle DCE$  and  $m(\angle ECX) = 62^\circ$ , Then :  $m(\angle A) =$  .....

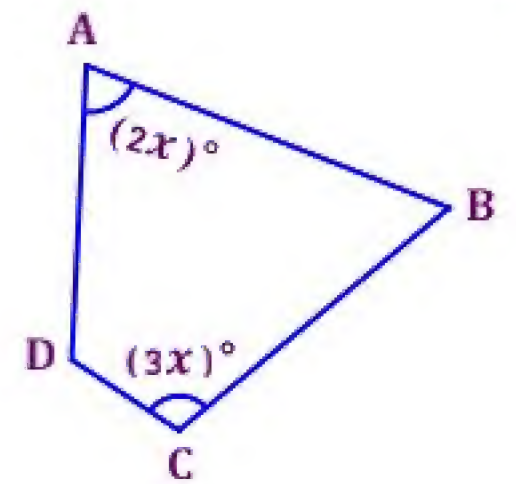
- (a)  $62^\circ$  (b)  $128^\circ$   
(c)  $56^\circ$  (d)  $124^\circ$



31 in the opposite figure :

ABCD is a cyclic quadrilateral, in which :  $m(\angle A) = (2x)^\circ$ ,  $m(\angle C) = (3x)^\circ$ , then :  $x =$  ..... « R. sea 2016 »

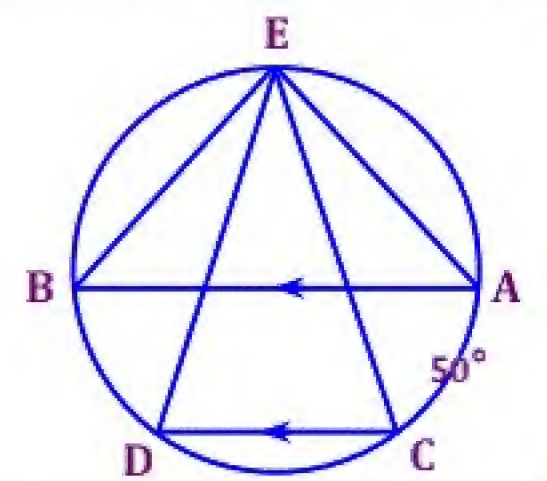
- (a)  $20^\circ$  (b)  $32^\circ$   
(c)  $32^\circ$  (d)  $36^\circ$



32 in the opposite figure :

$\overline{AB} \parallel \overline{CD}$ ,  $m(\widehat{AC}) = 50^\circ$ , Then :  $m(\angle BED) =$  .....

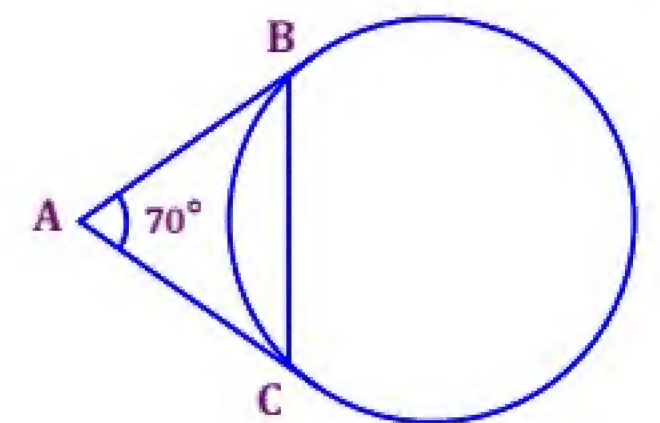
- (a)  $50^\circ$  (b)  $5^\circ$   
(c)  $25^\circ$  (d)  $20^\circ$



33 in the opposite figure :

$\overline{AB}$  and  $\overline{AC}$  Two tangents to the circle M from the point A,  $m(\angle BAM) = 70^\circ$ , Then :  $m(\widehat{AC}) =$  .....

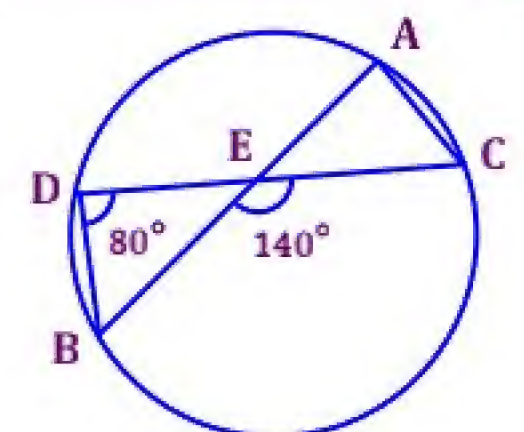
- (a)  $150^\circ$  (b)  $110^\circ$   
(c)  $100^\circ$  (d)  $90^\circ$



34 in the opposite figure :

$\overline{AB} \cap \overline{DC} = \{E\}$ ,  $m(\angle CEM) = 140^\circ$ ,  $m(\angle CDB) = 80^\circ$ , Then :  $m(\angle ACD) =$  .....

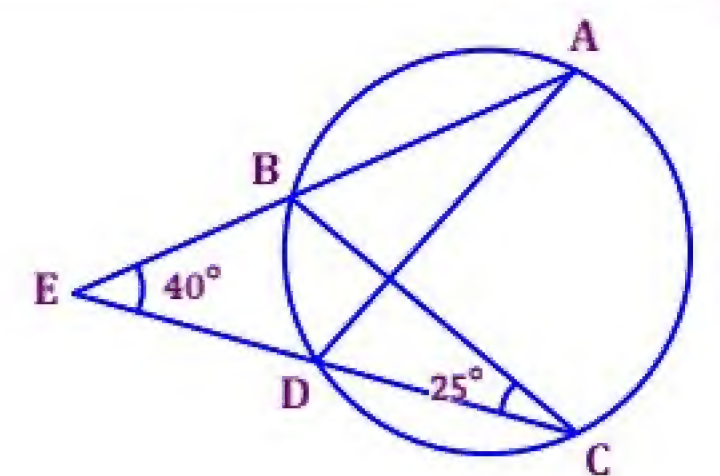
- (a)  $30^\circ$  (b)  $40^\circ$   
(c)  $50^\circ$  (d)  $60^\circ$



35 in the opposite figure :

$\overline{AB} \cap \overline{DC} = \{E\}$ ,  $m(\angle E) = 40^\circ$ ,  $m(\angle DCB) = 25^\circ$ , Then :  $m(\angle ABC) =$  .....

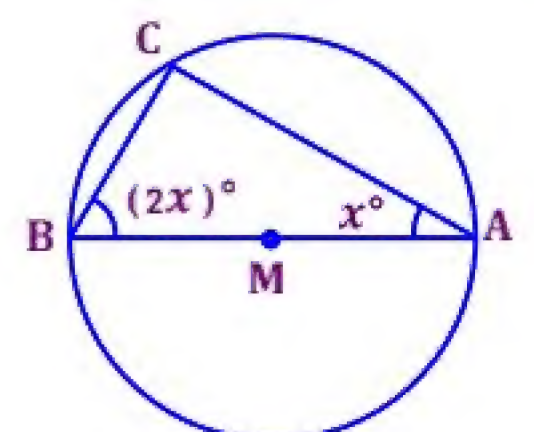
- (a)  $50^\circ$  (b)  $80^\circ$   
(c)  $25^\circ$  (d)  $65^\circ$



36 in the opposite figure :

$\overline{AB}$  is a diameter in circle M,  $m(\angle CAB) = x^\circ$ ,  $m(\angle CBA) = x^\circ$ , Then :  $x =$  .....

- (a)  $20^\circ$  (b)  $30^\circ$   
(c)  $40^\circ$  (d)  $60^\circ$

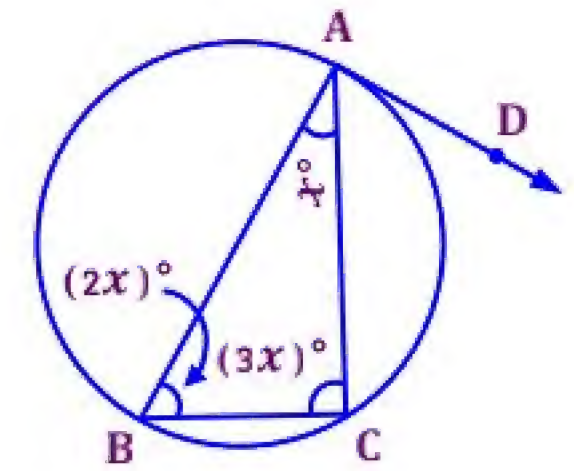




37 in the opposite figure :

$\overline{AD}$  touches the circle M at A ,  $m(\angle CAB) = x^\circ$  ,  $m(\angle CBA) = 2x^\circ$  ,  $m(\angle ACB) = 3x^\circ$  , Then :  $m(\angle DAC) =$  .....

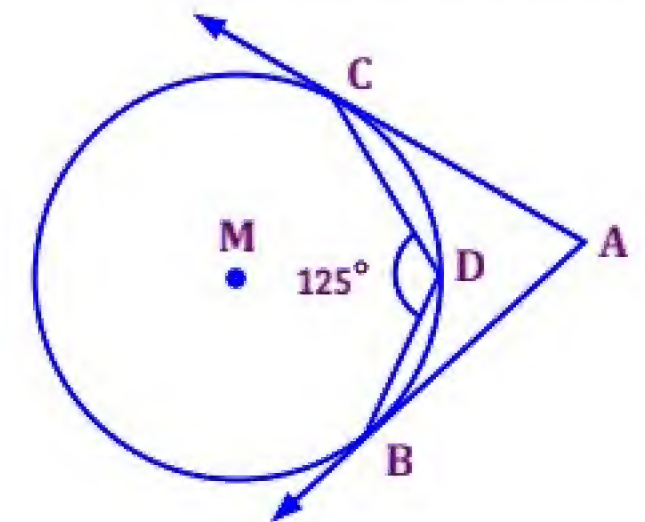
- (a)  $20^\circ$  (b)  $40^\circ$   
(c)  $60^\circ$  (d)  $80^\circ$



38 in the opposite figure :

$\overline{AB}$  and  $\overline{AC}$  Two tangents to the circle M from the point A ,  $m(\angle CDB) = 125^\circ$  , Then :  $m(\angle A) =$  .....

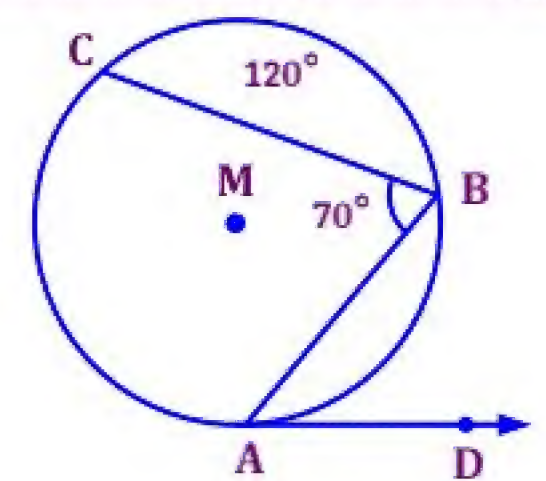
- (a)  $50^\circ$  (b)  $60^\circ$   
(c)  $70^\circ$  (d)  $80^\circ$



39 in the opposite figure :

$\overline{AD}$  touches the circle M at A ,  $m(\angle CAB) = 70^\circ$  ,  $m(\widehat{BC}) = 120^\circ$  , Then :  $m(\angle A) =$  .....

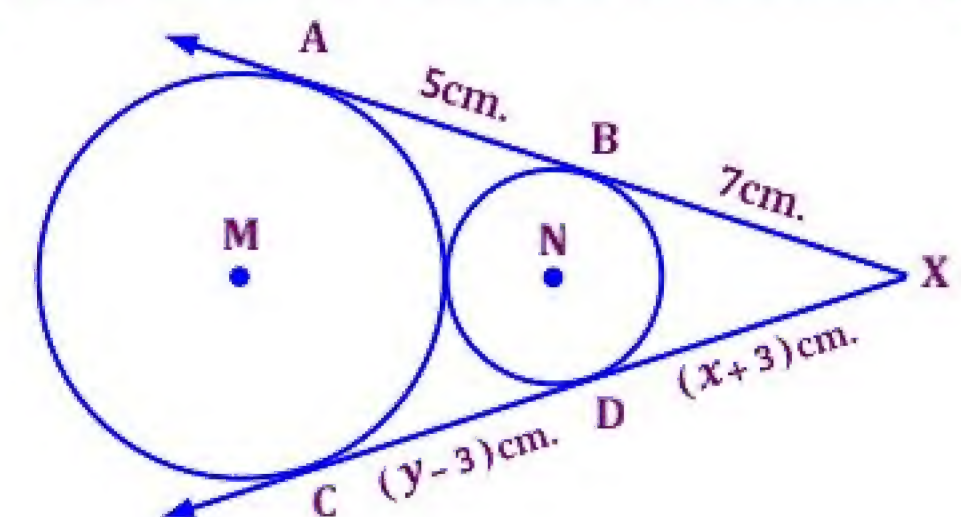
- (a)  $50^\circ$  (b)  $60^\circ$   
(c)  $35^\circ$  (d)  $70^\circ$



40 in the opposite figure :

$\overline{AB}$  and  $\overline{AC}$  Two to the two circles M and N from the point X  
 $BX = 7$  cm. ,  $DX = (x + 3)$  cm. ,  $AB = 5$  cm. and  $CD = (y - 3)$  cm. , Then :  $x + y =$  .....

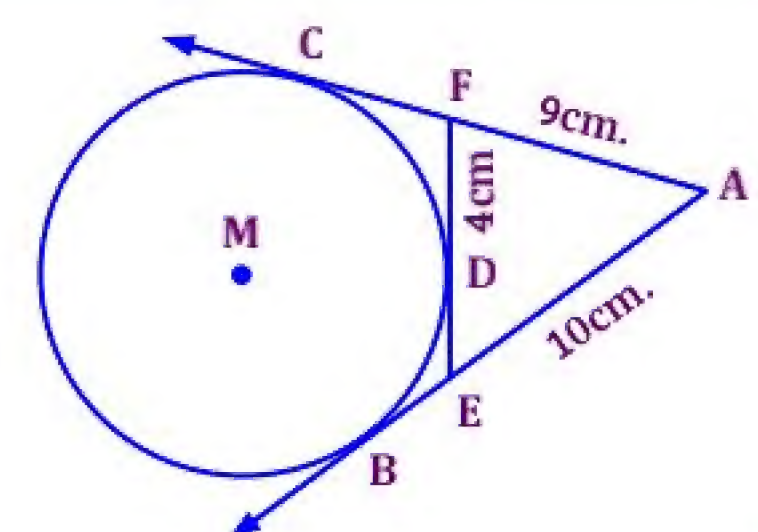
- (a) 10 (b) 11  
(c) 12 (d) 14



41 in the opposite figure :

$\overline{AB}$  and  $\overline{AC}$  Two tangents to the circle M from the point A  
 $FD = 4$  cm. ,  $FA = 9$  cm. ,  $AE = 10$  cm. , Then :  $ED =$  ..... cm.

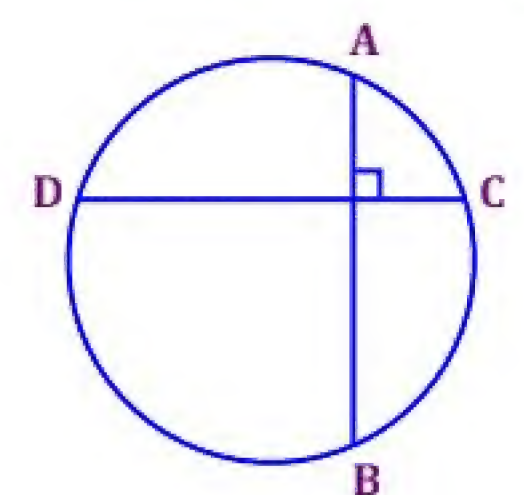
- (a) 3 (b) 4  
(c) 5 (d) 6



42 in the opposite figure :

$\overline{AB}$  and  $\overline{DC}$  are two radii perpendicular in the circle M ,  
 , Then :  $m(\widehat{AC}) + m(\widehat{BD}) =$  .....

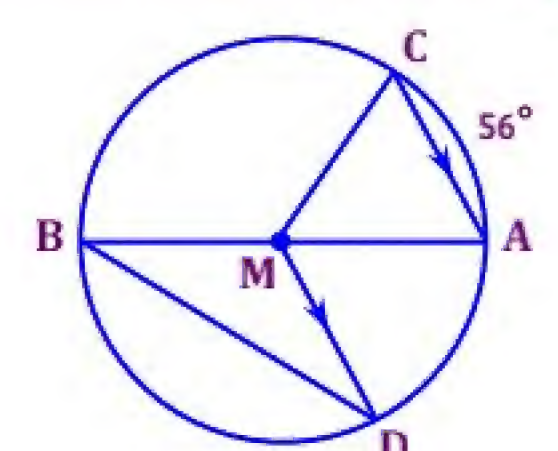
- (a)  $45^\circ$  (b)  $90^\circ$   
(c)  $180^\circ$  (d)  $270^\circ$



43 in the opposite figure :

$\overline{AC} \parallel \overline{DM}$  and  $m(\widehat{AC}) = 56^\circ$  , then :  $m(\angle ABD) =$  .....

- (a)  $28^\circ$  (b)  $56^\circ$   
(c)  $62^\circ$  (d)  $31^\circ$



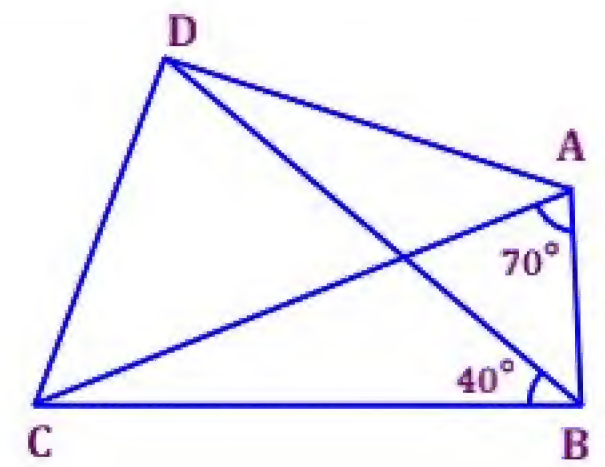


44 in the opposite figure :

ABCD is a cyclic quad , in which :  $m(\angle BAC) = 70^\circ$  ,  $(\angle DBC) = 40^\circ$

, Then :  $m(\angle DCB) =$  .....

- (a)  $30^\circ$  (b)  $40^\circ$   
(c)  $70^\circ$  (d)  $110^\circ$



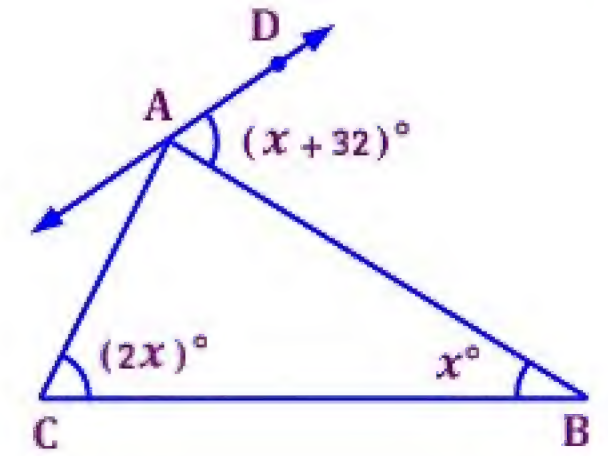
45 in the opposite figure :

If :  $\overline{AD}$  touches the circle whose passes through the vertices of the triangle ABC

at A ,  $m(\angle DAB) = (x + 32)^\circ$  ,  $m(\angle ACB) = (2x)^\circ$  ,  $m(\angle ABC) = x^\circ$

, Then :  $m(\angle BAC) =$  .....

- (a)  $32^\circ$  (b)  $64^\circ$   
(c)  $84^\circ$  (d)  $42^\circ$

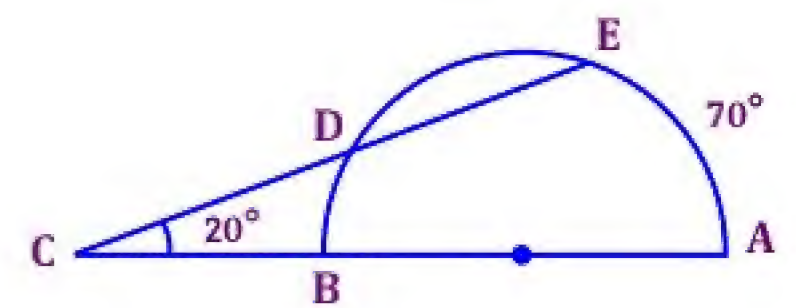


46 in the opposite figure :

$\overline{AB}$  is a diameter in circle M ,  $m(\angle ECA) = 20^\circ$  ,  $m(\widehat{AE}) = 70^\circ$

, Then :  $m(\widehat{DE}) =$  .....

- (a)  $70^\circ$  (b)  $80^\circ$   
(c)  $90^\circ$  (d)  $110^\circ$

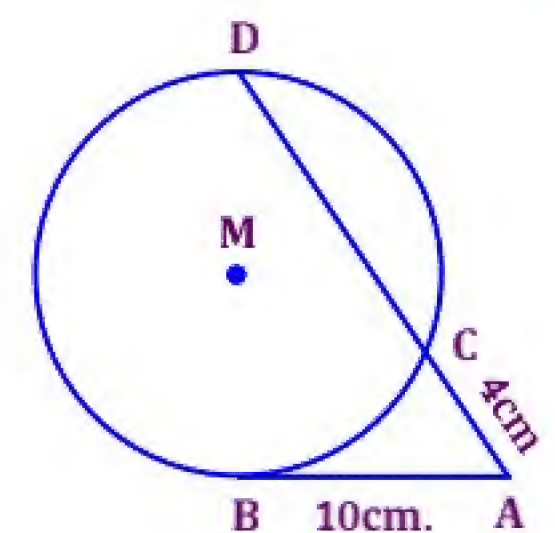


47 in the opposite figure :

$\overline{AB}$  is a tangent to the circle at B ,  $AB = 10$  cm. ,  $AC = 4$  cm.

, Then :  $DC =$  ..... cm.

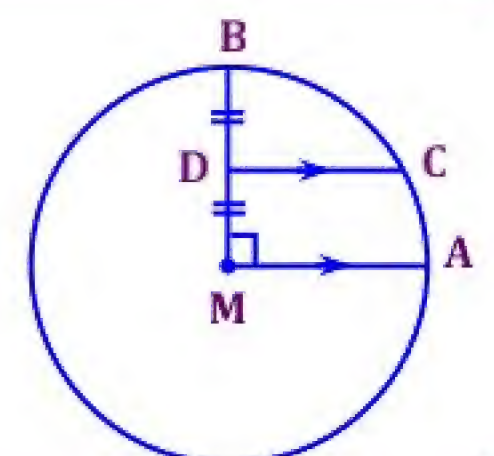
- (a) 4 (b) 6  
(c) 9 (d) 10



48 in the opposite figure :

$\overline{DC} \parallel \overline{AM}$  and D is a midpoint of  $\overline{BM}$  , Then :  $m(\widehat{AC}) =$  .....

- (a)  $60^\circ$  (b)  $30^\circ$   
(c)  $45^\circ$  (d)  $90^\circ$

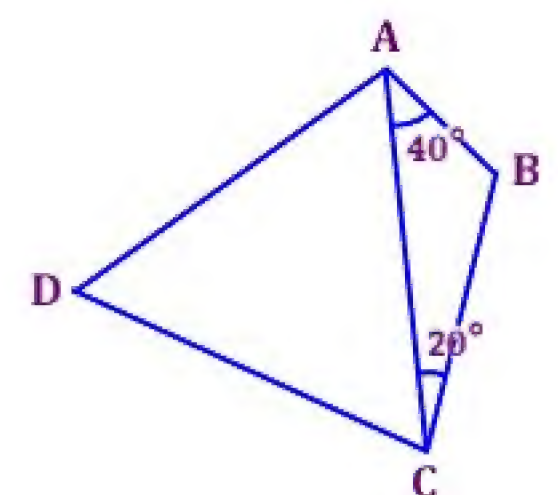


49 in the opposite figure :

ABCD is a cyclic quad , in which :  $m(\angle BAC) = 40^\circ$  ,  $(\angle ACB) = 20^\circ$

, Then :  $m(\angle D) =$  .....

- (a)  $20^\circ$  (b)  $40^\circ$   
(c)  $60^\circ$  (d)  $120^\circ$





## SECOND: GEOMETRY

Choose the correct answer:

- (1) The line of centers of two intersecting circles is perpendicular to the common ..... and bisect it.
- a** diameter      **b** tangent      **c** chord      **d** arc
- (2) The line of centers of two intersecting circles is the axis of symmetry of the common .....
- a** diameter      **b** tangent      **c** chord      **d** arc
- (3) The measure of the inscribed angle drawn in a quarter of a circle = .....°
- a** 135      **b** 120      **c** 90      **d** 45
- (4) The center of the inscribed circle of triangle is the intersection point of .....
- a** medians      **c** altitudes  
**b** axes of its sides      **d** bisectors of its angles

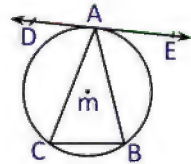


- (5) The circumference of a circle is  $8\pi$  cm and a straight line is on a distance 3 cm from its center, then L is ..... the circle.  
 (a) outside (b) secant to (c) tangent to (d) otherwise

- (6) If ABCD is a cyclic quadrilateral and  $m(\angle A) = 3m(\angle C)$ , then  $m(\angle A) = \dots\dots\dots^\circ$   
 (a) 180 (b) 135 (c) 90 (d) 45

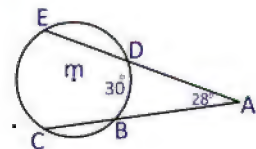
- (7) M and N are two intersecting circles of radii lengths 6 cm and 4 cm, then  $MN \in \dots\dots\dots$   
 (a)  $]10, \infty[$  (b)  $]2, 10[$  (c)  $]0, 2[$  (d)  $]4, 6[$

- (8) In the opposite figure:  
 $\overleftrightarrow{ED}$  is a tangent,  $m(\angle DAB) = 110^\circ$ ,  
 then  $m(\angle C) \dots\dots\dots$



- (a) 35 (b) 55 (c) 110 (d) 70
- (9) A circle of radius length 5 cm,  $\overline{AB}$  is a chord of length 8 cm, then the distance between the chord and the center = ..... cm.  
 (a) 3 (b) 6 (c) 8 (d) 10

- (10) From the opposite figure:  
 $m(\text{arc } EC) = \dots\dots\dots^\circ$



- (a) 56 (b) 30 (c) 86 (d) 28

- (11) From the opposite figure:  
 $m(\angle A) = \dots\dots\dots^\circ$



- (a) 20 (b) 40 (c) 50 (d) 80
- (12) The measure of the central angle in a circle ..... the measure of the inscribed angle subtended by the same arc.  
 (a) supplements (b) equal (c) half (d) double



- (13) The length of an arc which represents a semicircle = .....
- a**  $\pi r$       **b**  $2 \pi r$       **c**  $\frac{1}{2} \pi r$       **d**  $\frac{1}{4} \pi r$
- (14) If AB = 6 cm, then the number of circles which passes through A and B of radius length 3 cm is .....
- a** 0      **b** 1      **c** 2      **d** infinite
- (15) If AB = 5 cm, then the number of circles which passes through A and B of radius length 3 cm is .....
- a** 0      **b** 1      **c** 2      **d** infinite
- (16) If AB = 8 cm, then the number of circles which passes through A and B of radius length 3 cm is .....
- a** 0      **b** 1      **c** 2      **d** infinite
- (17) The number of common tangents of two distant circles is .....
- a** 1      **b** 2      **c** 3      **d** 4
- (18) If the longest chord in a circle is 12 cm, then its circumference = ..... cm
- a**  $6 \pi$       **b**  $12 \pi$       **c**  $24 \pi$       **d**  $144 \pi$
- (19) If the lengths of the radii of the two circles M and N are 6 cm, 8 cm and MN = 14 cm, then the two circles are .....
- a** intersecting      **c** touching externally  
**b** distant      **d** one inside the other
- (20) The inscribed angle in a semicircle is .....
- a** acute      **b** straight      **c** right      **d** obtuse
- (21) A chord of length 8 cm drawn in a circle of diameter length 10 cm, then the distance between the chord and the center is ..... cm.
- a** 3      **b** 4      **c** 5      **d** 6



- (22) Number of tangents of two touching internally circles is .....  
**a** 0                      **b** 1                      **c** 2                      **d** 3
- (23) If ABCD is a cyclic quadrilateral and  $m(\angle A) = 2m(\angle C)$ , then  $m(\angle A) = \dots\dots\dots^\circ$   
**a** 30                      **b** 60                      **c** 90                      **d** 120
- (24) If the lengths of radii of two circles M and N are 6 cm, 3 cm and  $MN = 2$  cm, then the two circles are .....  
**a** intersecting                      **c** one inside the other  
**b** distant                      **d** touching externally
- (25) Circle of diameter length  $2x$  cm, a straight line of distance  $x+1$  cm from its center, then the straight line is ..... circle.  
**a** tangent to the                      **c** secant to the  
**b** axis of symmetry of the                      **d** outside the
- (26) Number of common tangents of a two concentric circles is .....  
**a** 3                      **b** 2                      **c** 1                      **d** 0
- (27) The measure of the inscribed angle in a semicircle = ..... $^\circ$   
**a** 360                      **b** 180                      **c** 120                      **d** 90
- (28) If the lengths of radii of two circles M and N are 9 cm, 4 cm and  $MN = 5$  cm, then the two circles are .....  
**a** intersecting                      **c** touching internally  
**b** distant                      **d** touching externally
- (29) The centers of circles which passes through the two points A and B lies on .....  
**a**  $\overline{AB}$                       **c** axis of  $\overline{AB}$   
**b** midpoint of  $\overline{AB}$                       **d** perpendicular to  $\overline{AB}$  at B
- (30) The measure of the inscribed angle in a third of a circle = ..... $^\circ$   
**a** 360                      **b** 180                      **c** 120                      **d** 90

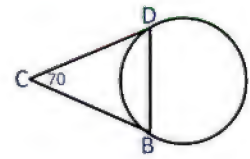


(31) The measure of the inscribed angle in a quarter of a circle is .....°

- a** 45                      **b** 90                      **c** 135                      **d** 145

(32) From the opposite figure:

$$m(\text{arc } BD) = \dots\dots\dots^\circ$$



- a** 55                      **b** 90                      **c** 180                      **d** 110

(33) The length of an arc which represents a quarter of a circle of radius length  $r$  cm is ..... cm

- a**  $4\pi r$                       **b**  $2\pi r$                       **c**  $\pi r$                       **d**  $\frac{1}{2}\pi r$

(34) One of the following identifies a unique circle .....

- a** length of radius and a point                      **c** one point  
**b** two points                      **d** center and a point

(35) Circle of diameter length 6 cm, a straight line of distance 6 cm from its center, then the straight line is .....

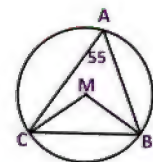
- a** outside the circle                      **c** secant to the circle  
**b** tangent to the circle                      **d** passes through the center

(36) If ABCD is a cyclic quadrilateral,  $m(\angle A) = 90^\circ$ , then the diameter of the circle is .....

- a**  $\overline{AB}$                       **b**  $\overline{AC}$                       **c**  $\overline{AD}$                       **d**  $\overline{BD}$

(37) From the opposite figure:

$$m(\angle MCB) = \dots\dots\dots^\circ$$



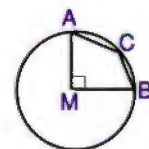
- a** 110                      **b** 35                      **c** 45                      **d** 55

(38) Number of axes of symmetry of two congruent circles and touching externally is .....

- a** 3                      **b** 2                      **c** 1                      **d** infinite



- (39) Number of axes of symmetry of two touching externally circles is .....
- a** 3                      **b** 2                      **c** 1                      **d** infinite
- (40) Number of common tangents of two touching externally circles is .....
- a** 3                      **b** 2                      **c** 1                      **d** infinite
- (41) Two touching circles of radii lengths 5 cm and 8 cm, then the distance between their centers  $\in$  .....
- a**  $]13,3[$                       **b**  $]3,13[$                       **c**  $R-[3,13]$                       **d**  $\{3,13\}$
- (42) Two intersecting circles of radii lengths 5 cm and 3 cm, then the distance between their centers  $\in$  .....
- a**  $]8,\infty[$                       **b**  $]2,\infty[$                       **c**  $]0,2]$                       **d**  $]2,8[$
- (43) We can't draw a circle passing through the vertices of .....
- a** triangle                      **b** rectangle                      **c** rhombus                      **d** square
- (44) The minor arc in the circle is opposite to ..... inscribed angle.
- a** an acute                      **b** an obtuse                      **c** a right                      **d** a reflex
- (45) The radius length of the smallest angle passing through the endpoints of a line segment ..... half of its length.
- a** less than                      **b** more than                      **c** equals                      **d** double
- (46) The two tangents to a circle drawn from the endpoints of its diameter are .....
- a** parallel                      **b** equal                      **c** coincident                      **d** intersecting
- (47) From the opposite figure:  
 $m(\angle ACB) = \dots\dots\dots^\circ$
- a** 45                      **b** 110                      **c** 135                      **d** 270





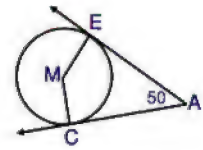
(48) The measure of an arc which represents a third of a circle is .....°

- a** 60                      **b** 90                      **c** 120                      **d** 240

(49) From the opposite figure:

$$m(\text{arc } EC) = \text{.....}^\circ$$

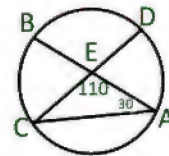
- a** 100                      **b** 120                      **c** 130                      **d** 50



(50) From the opposite figure:

$$m(\text{arc } DA) = \text{.....}^\circ$$

- a** 40                      **b** 55                      **c** 80                      **d** 110



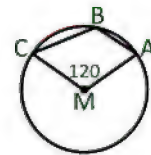
(51) If  $AB = 6$  cm, then the area of the smallest circle passing through A and B is .....  $\text{cm}^2$ .

- a**  $3\pi$                       **b**  $6\pi$                       **c**  $8\pi$                       **d**  $9\pi$

(52) From the opposite figure:

$$m(\angle ABC) = \text{.....}^\circ$$

- a** 60                      **b** 120                      **c** 240                      **d** 360



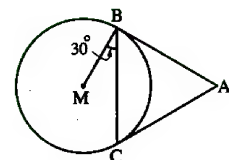
(53) The center of the circumcircle of a triangle is the intersection point of .....

- a** its medians                      **c** its altitudes  
**b** axes of its sides                      **d** bisectors of its angles

(54) From the opposite figure:

$$m(\angle BAC) = \text{.....}^\circ$$

- a** 90                      **b** 60                      **c** 30                      **d** 15



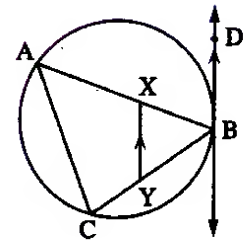


## Essay Problems:

(1)

**In the opposite figure :**

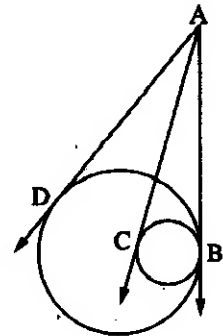
ABC is a triangle inscribed in a circle

,  $\overrightarrow{BD}$  is a tangent to the circle at B,  $X \in \overline{AB}$  ,  $Y \in \overline{BC}$  where  $\overline{XY} \parallel \overline{BD}$ **Prove that :** AXYC is a cyclic quadrilateral.

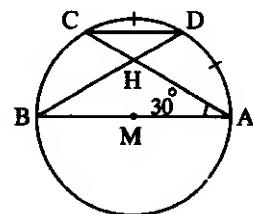
(2)

**In the opposite figure :**

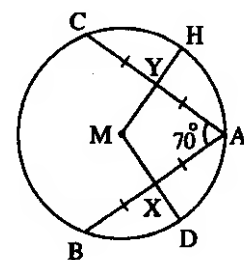
Two circles are touching internally at B

,  $\overrightarrow{AB}$  is a common tangent,  $\overrightarrow{AC}$  is a tangent to the smaller circle at C,  $\overrightarrow{AD}$  is a tangent to the greater circle at D,  $AC = 15$  cm. ,  $AB = (2x - 3)$  cm.and  $AD = (y - 2)$  cm.**Find :** The value of each of  $x$  and  $y$ 

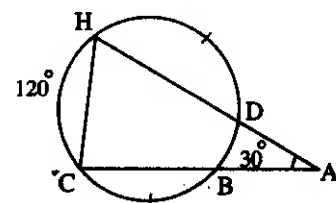
(3)

**In the opposite figure :** $\overline{AB}$  is a diameter in the circle M,  $C \in$  the circle M ,  $m(\angle CAB) = 30^\circ$ , D is midpoint of  $\widehat{AC}$  ,  $\overline{DB} \cap \overline{AC} = \{H\}$ (1) **Find :**  $m(\angle BDC)$  and  $m(\widehat{AD})$ (2) **Prove that :**  $\overline{AB} \parallel \overline{DC}$ 

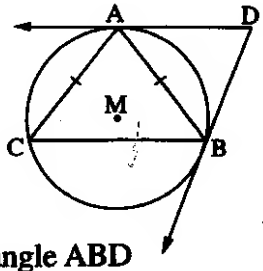
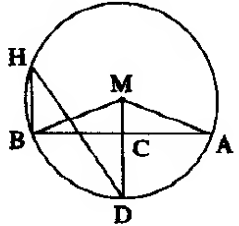
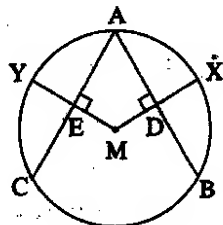
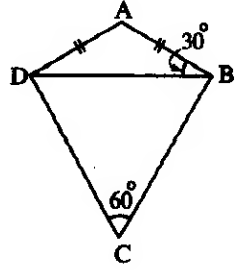
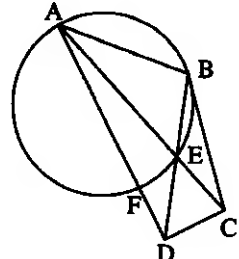
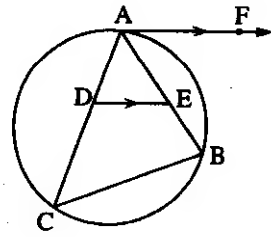
(4)

**In the opposite figure :** $\overline{AB}$  and  $\overline{AC}$  are two chords equal in length in circle M, X is the midpoint of  $\overline{AB}$  , Y is the midpoint of  $\overline{AC}$ ,  $m(\angle CAB) = 70^\circ$ (1) **Calculate :**  $m(\angle DMH)$ (2) **Prove that :**  $XD = YH$ 

(5)

**In the opposite figure :** $m(\angle A) = 30^\circ$  ,  $m(\widehat{HC}) = 120^\circ$ ,  $m(\widehat{BC}) = m(\widehat{DH})$ (1) **Find :**  $m(\widehat{BD})$  the minor(2) **Prove that :**  $AB = AD$ 



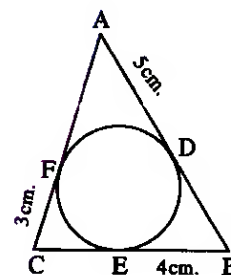
(6)	State two cases of a cyclic quadrilateral.
(7)	<p><b>In the opposite figure :</b>  <math>\overrightarrow{DA}</math> and <math>\overrightarrow{DB}</math> are two tangents of the circle M  and <math>AB = AC</math>  <b>Prove that :</b>  <math>\overrightarrow{AC}</math> is a tangent to the circle passing through the vertices of the triangle ABD</p> 
(8)	<p><b>In the opposite figure :</b>  C is the midpoint of <math>\overline{AB}</math>, <math>\overline{MC} \cap</math> the circle M = {D}  , <math>m(\angle MAB) = 20^\circ</math>  <b>Find :</b> <math>m(\angle BHD)</math> and <math>m(\widehat{ADB})</math></p> 
(9)	<p><b>In the opposite figure :</b>  <math>AB = AC</math> , <math>\overline{MD} \perp \overline{AB}</math> ,  <math>\overline{ME} \perp \overline{AC}</math>  <b>Prove that :</b> <math>XD = YE</math></p> 
(10)	<p><b>In the opposite figure :</b>  ABCD is a quadrilateral in which <math>AB = AD</math> ,  <math>m(\angle ABD) = 30^\circ</math> ,  <math>m(\angle C) = 60^\circ</math>  <b>Prove that :</b> ABCD is a cyclic quadrilateral.</p> 
(11)	<p><b>In the opposite figure :</b>  <math>\overline{BC}</math> is a tangent at B ,  E is the midpoint of <math>\widehat{BF}</math>  <b>Prove that :</b>  ABCD is a cyclic quadrilateral.</p> 
(12)	<p><b>In the opposite figure :</b>  <math>\overrightarrow{AF}</math> is a tangent to the circle at A , <math>\overrightarrow{AF} \parallel \overrightarrow{DE}</math>  <b>Prove that :</b>  DEBC is a cyclic quadrilateral.</p> 



(13)

**In the opposite figure :**

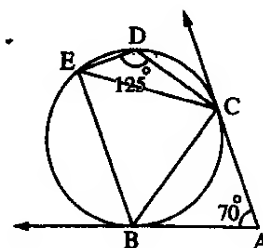
A circle is drawn touches the sides of a triangle  $ABC$ ,  $\overline{AB}$ ,  $\overline{BC}$ ,  $\overline{AC}$  at  $D$ ,  $E$ ,  $F$ ,  $AD = 5$  cm,  $BE = 4$  cm.,  $CF = 3$  cm.

Find the perimeter of  $\Delta ABC$ 

(14)

**In the opposite figure :**

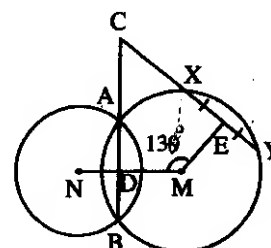
$\overline{AB}$ ,  $\overline{AC}$  are two tangents to the Circle at  $B$ ,  $C$ ,  
 $m(\angle A) = 70^\circ$ ,  
 $m(\angle CDE) = 125^\circ$

**Prove that :**(1)  $CB = CE$ (2)  $\overline{AC} \parallel \overline{BE}$ 

(15)

**In the opposite figure :**

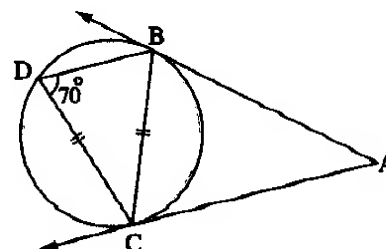
If  $E$  is the midpoint of  $\overline{XY}$   
 $m(\angle EMN) = 130^\circ$   
**then find :**  $m(\angle C)$



(16)

**In the opposite figure :**

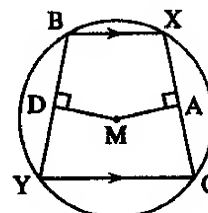
If  $\overline{AB}$ ,  $\overline{AC}$  are two tangents to the circle at  $B$ ,  $C$   
 $m(\angle D) = 70^\circ$ ,  $CB = CD$

(1) **Find :**  $m(\angle A)$ (2) **Prove that :**  $\overline{BD} \parallel \overline{AC}$ 

(17)

**Complete :** The measure of the inscribed angle equals ..... the measure of the central angle ..... by the same arc.

(18)

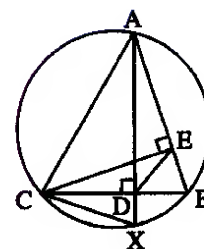
**In the opposite figure :** $\overline{XB} \parallel \overline{CY}$ ,  $\overline{MA} \perp \overline{XC}$  $\overline{MD} \perp \overline{BY}$ **Prove that :**  $MA = MD$ 



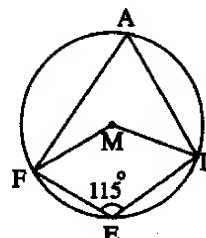
(19)

**In the opposite figure :** $\overline{CE} \perp \overline{AB}$  ,  $\overline{AD} \perp \overline{BC}$  and intersects the circle at X**Prove that :**

(1) AEDC is a cyclic quadrilateral.

(2)  $\overline{CB}$  bisects  $\angle ECX$ 

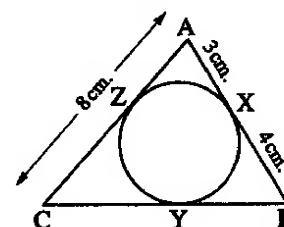
(20)

**In the opposite figure :**If  $m(\angle DEF) = 115^\circ$ , then find :  $m(\angle DMF)$ 

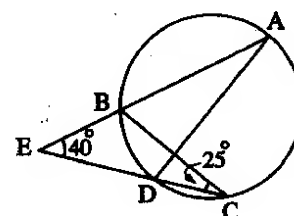
(21)

**In the opposite figure :**

Inscribed circle of the triangle ABC touches its sides at X , Y and Z

If  $AX = 3$  cm. ,  $XB = 4$  cm. ,  $AC = 8$  cm.**Find :** The length of  $\overline{BC}$ 

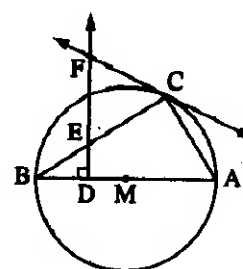
(22)

**In the opposite figure :** $\overline{AB} \cap \overline{CD} = \{E\}$  ,  $m(\angle C) = 25^\circ$ ,  $m(\angle E) = 40^\circ$ **Find :**  $m(\angle ADC)$ 

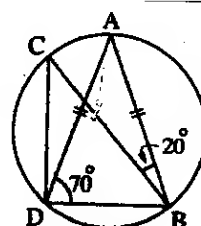
(23)

**In the opposite figure :** $\overline{AB}$  is a diameter in the circle M,  $\overline{CF}$  is a tangent to the circle at C,  $\overline{DF} \perp \overline{AB}$  and intersects  $\overline{BC}$  at E**Prove that :**

(1) ADEC is a cyclic quadrilateral.

(2)  $\triangle FCE$  is an isosceles triangle.

(24)

**In the opposite figure :** $AB = AD$ ,  $m(\angle ABC) = 20^\circ$ ,  $m(\angle ADB) = 70^\circ$ **Find :**  $m(\angle C)$  ,  $m(\angle BDC)$ 



(25)

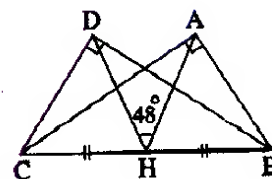
**In the opposite figure :**

$$m(\angle BAC) = m(\angle BDC) = 90^\circ$$

, H is the midpoint of  $\overline{BC}$  and  $m(\angle AHD) = 48^\circ$

(a) **Prove that :** ABCD is a cyclic quadrilateral.

(e) **Find :**  $m(\angle ABD)$



(26)

Using your geometric tools , draw  $\overline{AB}$  with a length of 4 cm. , then draw a circle passing through the two points A and B whose radius length is 3 cm.

What are the possible solutions ? (Don't remove the arcs)

(27)

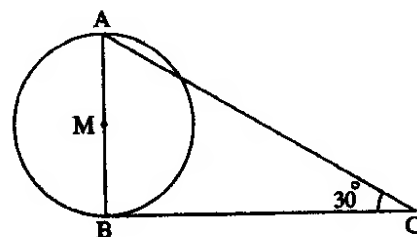
**In the opposite figure :**

A circle M of circumference 44 cm.

,  $\overline{AB}$  is a diameter ,  $\overline{BC}$  is a tangent at B

and  $m(\angle ACB) = 30^\circ$

**Find :** The length of  $\overline{BC}$  ( $\pi = \frac{22}{7}$ )

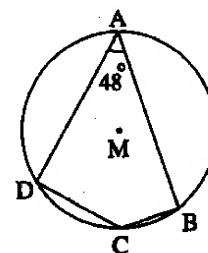


(28)

**In the opposite figure :**

If M is a circle ,  $m(\angle A) = 48^\circ$

**Find :**  $m(\widehat{BD})$  the major



(29)

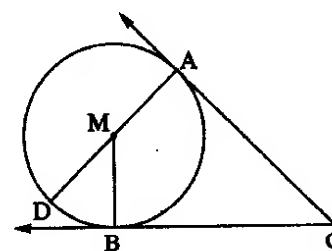
**In the opposite figure :**

$\overline{AD}$  is a diameter in a circle M

,  $\overrightarrow{CA}$  and  $\overrightarrow{CB}$  are two tangents to the circle M ,

touch it at A and B respectively.

**Prove that :**  $m(\angle DMB) = m(\angle ACB)$



(30)

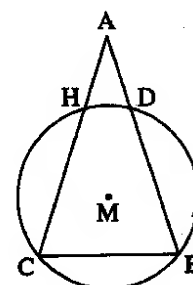
**In the opposite figure :**

ABC is a triangle in which  $AB = AC$

,  $\overline{BC}$  is a chord in the circle M

, if  $\overline{AB}$  and  $\overline{AC}$  cut the circle at D and H respectively.

**Prove that :**  $m(\widehat{DB}) = m(\widehat{HC})$

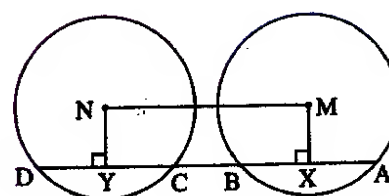




(31)

**In the opposite figure :**

M and N are two congruent circles

,  $AB = CD$ **Prove that :** The figure MXYN is a rectangle.

(32)

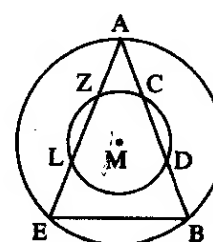
ABCD is a quadrilateral inscribed in a circle , H is a point outside the circle and  $\overrightarrow{HA}$  and  $\overrightarrow{HB}$  are two tangents to the circle at A and B , if  $m(\angle AHB) = 70^\circ$  and  $m(\angle ADC) = 125^\circ$  , **prove that :**

①  $AB = AC$ ②  $\overrightarrow{AC}$  is a tangent to the circle passing through the points A , B and H

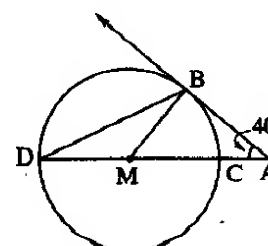
(33)

**In the opposite figure :**

Two concentric circles at M

,  $m(\angle ABE) = m(\angle AEB)$ **Prove that :**  $CD = ZL$ 

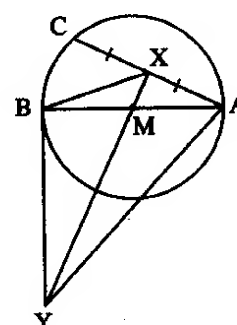
(34)

**In the opposite figure :** $\overrightarrow{AB}$  is a tangent to the circle M,  $m(\angle A) = 40^\circ$ **Find with proof :**  $m(\angle BDC)$ 

(35)

**In the opposite figure :** $\overline{AB}$  is a diameter in the circle M, X is the midpoint of  $\overline{AC}$  and  $\overline{XM}$  intersecting

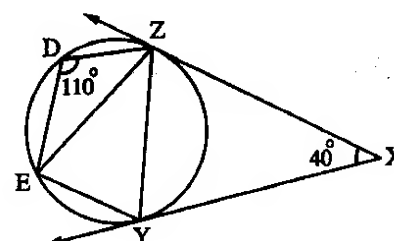
the tangent of the circle at B in Y

**Prove that :** The figure AXBY is a cyclic quadrilateral.

(36)

**In the opposite figure :**

$\overrightarrow{XY}$  and  $\overrightarrow{XZ}$  are two tangents to the circle at the two points Y and Z ,  $m(\angle X) = 40^\circ$

,  $m(\angle D) = 110^\circ$ **Prove that :**  $m(\angle ZYE) = m(\angle ZEY)$ 

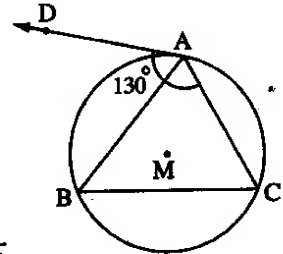


(37)

ABCD is a quadrilateral drawn in a circle ,  $E \in \overrightarrow{AB}$  ,  $E \notin \overline{AB}$   
 $m(\widehat{AB}) = 110^\circ$  ,  $m(\angle CBE) = 85^\circ$   
**Find with proof :**  $m(\angle BDC)$

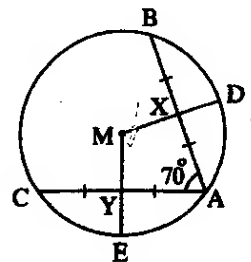
(38)

**In the opposite figure :**  
 $\overrightarrow{AD}$  is the tangent to the circle M at A  
 $m(\angle DAC) = 130^\circ$   
**Find with proof :**  $m(\angle B)$



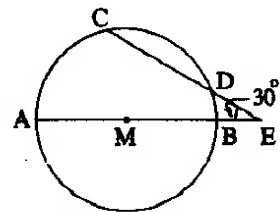
(39)

**In the opposite figure :**  
 $\overline{AB}$  and  $\overline{AC}$  are two chords equal in length at the circle M  
 $X$  is the midpoint of  $\overline{AB}$   
 $Y$  is the midpoint of  $\overline{AC}$  ,  $m(\angle A) = 70^\circ$   
**(1) Find :**  $m(\angle DME)$       **(2) Prove that :**  $XD = YE$



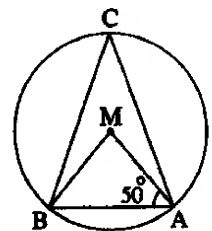
(40)

**In the opposite figure :**  
 $\overline{AB}$  is a diameter in the circle M  
 $\overrightarrow{AB} \cap \overrightarrow{CD} = \{E\}$  ,  $m(\angle E) = 30^\circ$  ,  $m(\widehat{AC}) = 80^\circ$   
**Find :**  $m(\widehat{CD})$



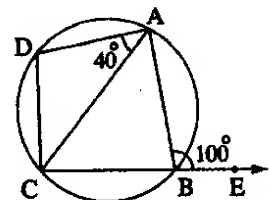
(41)

**In the opposite figure :**  
 $M$  is a circle ,  $m(\angle MAB) = 50^\circ$   
**Find :**  $m(\angle C)$



(42)

**In the opposite figure :**  
 $m(\angle ABE) = 100^\circ$   
 $m(\angle CAD) = 40^\circ$   
**Prove that :**  $\triangle DAC$  is an isosceles triangle.





(43)

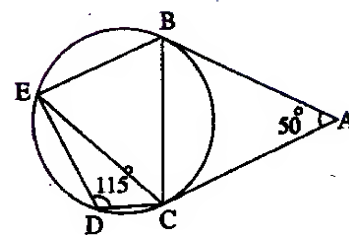
**In the opposite figure :**

$\overline{AB}$  and  $\overline{AC}$  are two tangent-segments  
to the circle at B and C

$m(\angle A) = 50^\circ$  ,  $m(\angle D) = 115^\circ$

**Prove that :** (1)  $\overrightarrow{BC}$  bisects  $\angle ABE$

(2)  $CB = CE$



(44)

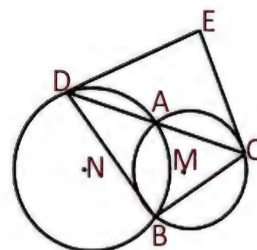
**In the opposite figure:**

M, N are two intersecting circles in A , B

$\overrightarrow{EC}$  is tangent to the circle M at C,

$\overrightarrow{DC}$  is tangent to the circle N at D,

Prove that: ECDB is cyclic quadrilateral

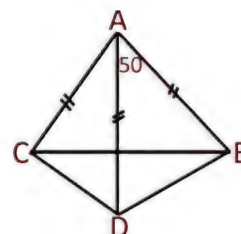


(45)

**In the opposite figure:**

$AB = AC = AD$ ,  $m(\angle BAD) = 50^\circ$

Find  $m(\angle BCD)$



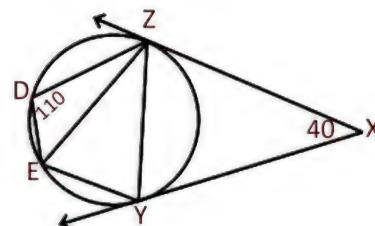
(46)

**In the opposite figure:**

$\overline{XY}$  ,  $\overline{XZ}$  are two tangents to the circle

$m(\angle YXZ) = 40^\circ$  ,  $m(\angle ZDE) = 110^\circ$

Prove that:  $ZE = ZY$

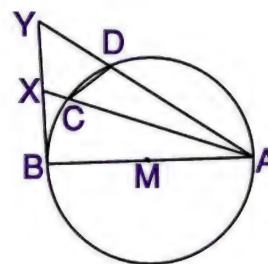


(47)

**In the opposite figure:**

$\overline{AB}$  is a diameter in circle M,  $\overline{YB}$  is tangent.

Prove that: DCXY is cyclic quadrilateral





PREP 3

FINAL REVISION

## SECOND GEOMETRY

Choose the correct answer:

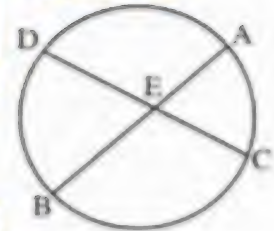
1) In the opposite figure:

$$m \widehat{(DB)} = 80^\circ,$$

$$m \widehat{(AC)} = 60^\circ, \text{ then}$$

$$m (\angle AEC) = \dots\dots\dots$$

(  $20^\circ$  or  $30^\circ$  or  $70^\circ$  or  $140^\circ$  )



2) The two tangents which are drawn from the two endpoints of a diameter of a circle are .....

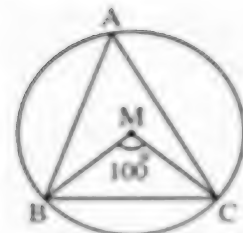
( parallel. or intersecting. or perpendicular. or coincide.)

3) In the opposite figure:

$$M \text{ is a circle , } m (\angle BMC) = 100^\circ$$

$$\text{, then } m (\angle BAC) = \dots\dots\dots$$

(  $150^\circ$  or  $100^\circ$  or  $50^\circ$  or  $25^\circ$  )

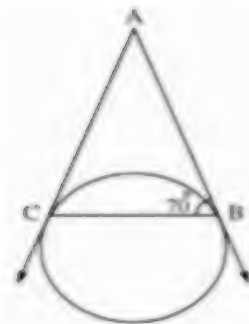


4) In the opposite figure:

$\overrightarrow{AB}$  and  $\overrightarrow{AC}$  are two tangents to the circle at B and C ,  $m (\angle ABC) = 70^\circ$

$$\text{, then } m (\angle A) = \dots\dots\dots$$

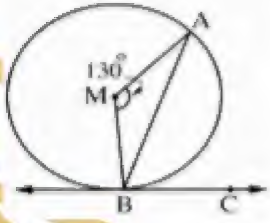
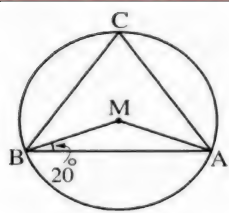
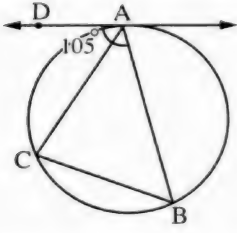
(  $140^\circ$  or  $70^\circ$  or  $40^\circ$  or  $35^\circ$  )



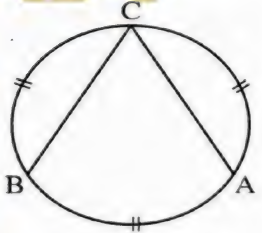
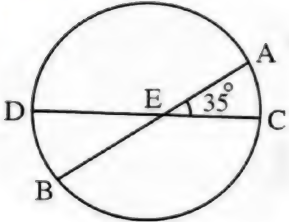
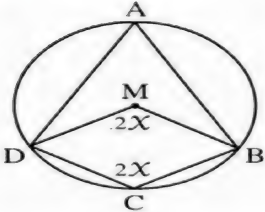
5) Sum of the measures of any two opposite angles in the cyclic quadrilateral equals .....

(  $90^\circ$  or  $180^\circ$  or  $270^\circ$  or  $360^\circ$  )



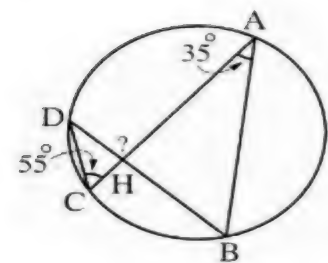
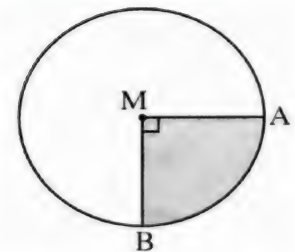
6)	Measure of an arc which represents $\frac{1}{3}$ of the measure of the circle equals = ..... ( 60° or 90° or 120° or 180° )
7)	In the opposite figure: $\overrightarrow{BC}$ is a tangent to the circle M at B if $m(\angle AMB) = 130^\circ$ , than $m(\angle ABC) = \dots\dots\dots$ ( 280° or 140° or 70° or 65° ) 
8)	The length of the arc which represents $\frac{1}{4}$ of circumference of a circle = ..... ( $2\pi r$ or $\pi r$ or $\frac{1}{2}\pi r$ or $\frac{1}{4}\pi r$ )
9)	In a cyclic quadrilateral, each two opposite angles are ..... equal or supplementary intersecting or corresponding
10)	If surface of circle M $\cap$ surface of circle N = $\emptyset$ , than the two circles are ..... intersecting or distant touching internally or touching externally
11)	In the opposite figure: Circle M, if $m(\angle MBA) = 20^\circ$ , then $m(\angle C) = \dots\dots\dots$ ( 120° or 70° or 40° or 30° ) 
12)	In the opposite figure: If $\overleftrightarrow{AD}$ is a tangent to the circle at A , $m(\angle DAB) = 105^\circ$ , then $m(\angle ACB) = \dots\dots\dots$ ( 75° or 60° or 50° or 35° ) 



13)	The number of common tangents for the two tangent circles externally is .....	(4 or 3 or 2 or infinite number)
14)	The figure which the circle doesn't passing through its vertices is .....	(square or rectangle or rhombus or triangle)
15)	In the opposite figure: $m(\angle C) = \dots\dots\dots$	 (45° or 50° or 30° or 60°)
16)	In the opposite figure: $(\angle AEC) = 35^\circ$ , then $m(\widehat{AC}) + m(\widehat{DB}) = \dots\dots\dots$	 (17.5° or 35° or 70° or 140°)
17)	The inscribed angle opposite to an arc greater than the semicircle is .....	(straight or acute or right or obtuse)
18)	In the opposite figure: If $m(\angle DMB) = m(\angle DCB) = 2x$ , then $m(\angle A) = \dots\dots\dots$	 (60° or 70° or 40° or 30°)
19)	The diameter length of a circle is 8 cm. if the straight line L is at a distance 4 cm. form the Centre , then the straight line L is .....	a secant to the circle. or outside the circle. a tangent to the circle. or an axis of symmetry to the circle.



20)	The measure of the exterior angle at any vertex of a cyclic quadrilateral vertices..... the measure of the opposite interior of the adjacent angle.  ( > or < or = or ≥ )
21)	The number of common tangents of two distant circles is ..... ( 4 or 3 or 2 or infinite )
22)	The length of the arc opposite to the inscribed angle of measure $.60^\circ = \dots\dots\dots$ Circumference of the circle. ( $\frac{1}{6}$ or $\frac{1}{3}$ or $\frac{1}{2}$ or otherwise )
23)	The inscribed angle drawn in a semicircle ..... (acute or obtuse or reflex or right )
24)	In the opposite figure: $\overline{MA}$ and $\overline{MB}$ two radii in a circle M , $\overline{MA} \perp \overline{MB}$ and the radius length is 7 cm. then the perimeter of the shaded part =.....cm. ( 14 or 21 or 38.5 or 25 )
25)	The measure of the circle with radius r is ..... ( $2\pi r$ or $180^\circ$ or $\pi r$ or $360^\circ$ )
26)	In the opposite figure: $m(\angle C) = 55^\circ$ , $m(\angle A) = 35^\circ$ , then $m(\angle AHD) = \dots\dots\dots$ ( $20^\circ$ or $90^\circ$ or $70^\circ$ or $110^\circ$ )
27)	The Centre of inscribed circle of a triangle is the intersection point of is .....  altitudes. or axes of symmetry of its sides. medians. or bisectors of its interior angles.





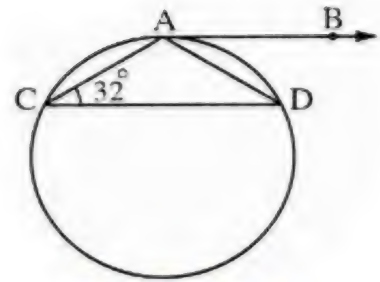
28) In the opposite figure:

$\overrightarrow{AB}$  is tangent to the circle,

$$m(\angle C) = 32^\circ$$

, then  $m(\angle BAD) = \dots\dots\dots$

(  $64^\circ$  or  $32^\circ$  or  $148^\circ$  or  $58^\circ$  )



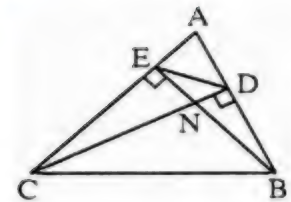
29) If M and N are two touching externally circles with radii lengths 9 cm. and r cm. respectively , if  $MN = 14$  cm. , then  $r = \dots\dots\dots$  cm.

(  $10$  or  $23$  or  $5$  or  $7$  )

30) In the opposite figure:

How many cyclic quadrilaterals?

(  $1$  or  $2$  or  $3$  or  $4$  )



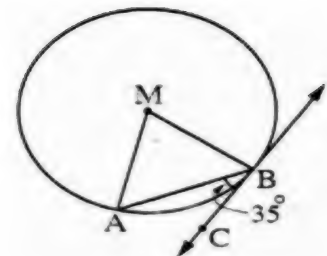
31) In the opposite figure:

$\overrightarrow{BC}$  is a tangent to the circle M

$$m(\angle ABC) = 35^\circ$$

, then  $m(\angle AMB) = \dots\dots\dots$

(  $105^\circ$  or  $120^\circ$  or  $70^\circ$  or  $60^\circ$  )

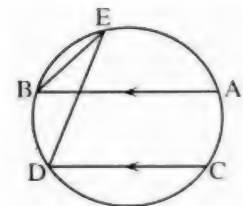


32) In the opposite figure:

$\overline{AB}$  and  $\overline{CD}$  are two parallel chords of a circle ,

$$m(\angle DEB) = 25^\circ, \text{ then } m(\widehat{AC}) = \dots\dots\dots$$

(  $100^\circ$  or  $75^\circ$  or  $50^\circ$  or  $25^\circ$  )



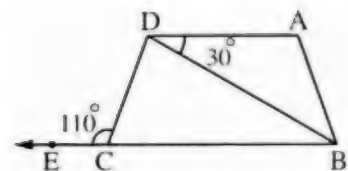
33) In the opposite figure:

ABCD is a cyclic quadrilateral ,

$$m(\angle ADB) = 30^\circ$$

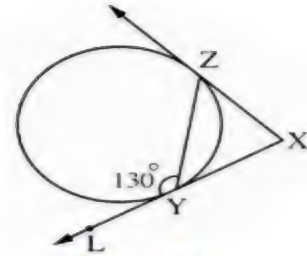
and  $m(\angle DCE) = 110^\circ$  , then  $m(\angle ABD) = \dots\dots\dots$

(  $30^\circ$  or  $40^\circ$  or  $60^\circ$  or  $70^\circ$  )



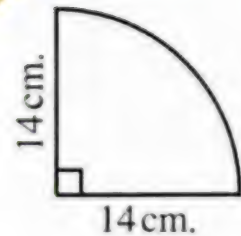


- 34) In the opposite figure:  
 $\overrightarrow{XZ}$ ,  $\overrightarrow{XL}$  are two tangents to the circle  
 at Y and Z,  
 $m(\angle LYZ) = 130^\circ$ , then  $m(\angle X) = \dots\dots\dots$   
 (50° or 65° or 80° or 100°)

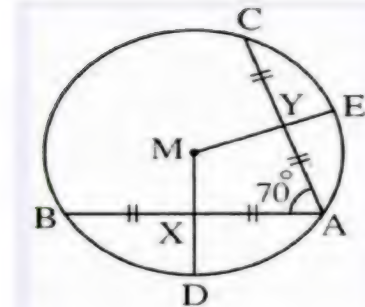


- 35) If the measures of the two arcs are equal in the same circle then  
 their chords are .....  
 intersecting. or parallel  
 perpendicular. or equal in length.

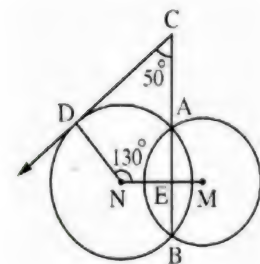
- 36) In the opposite figure:  
 A metallic wire is formed in the form of a quarter of  
 a circle of radius length 14 cm. as shown, then the  
 length of the wire = .....  
 where  $\pi = \frac{22}{7}$   
 (154 cm. or 50 cm. or 26 cm. or 22cm.)



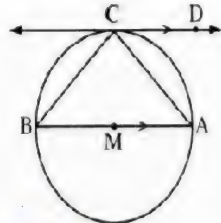
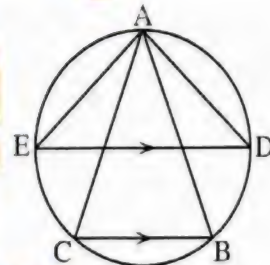
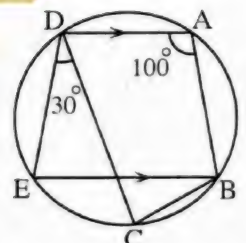
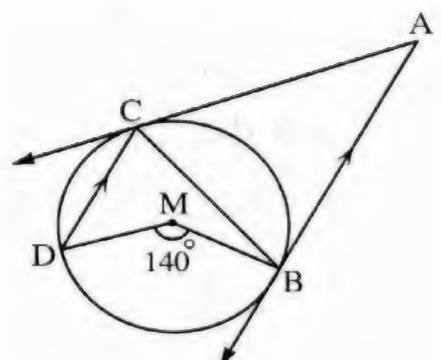
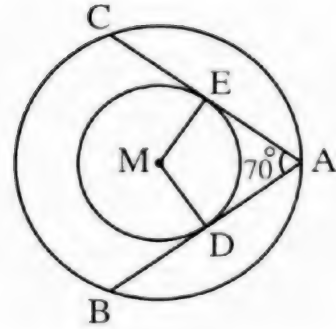
- 1) a) In the opposite figure:  
 $\overline{AB}$  and  $\overline{AC}$  are two equal chords in length  
 In the circle M, X is the midpoint of  
 $\overline{AB}$  and Y  
 Is the midpoint of  $\overline{AC}$ ,  $m(\angle CAB) = 70^\circ$   
 (1) Calculate :  $m(\angle DME)$   
 (2) Prove that:  $XD = YE$



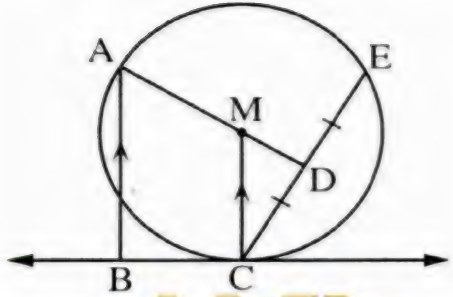
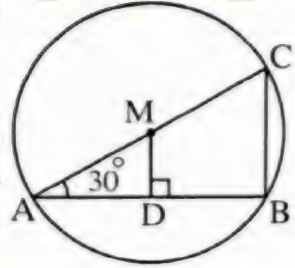
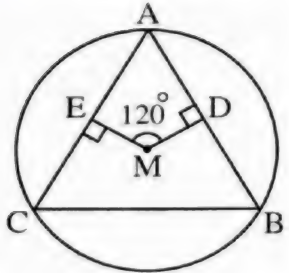
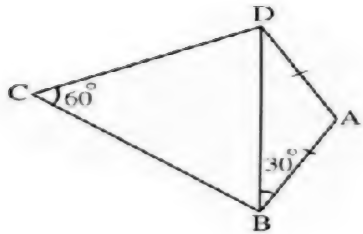
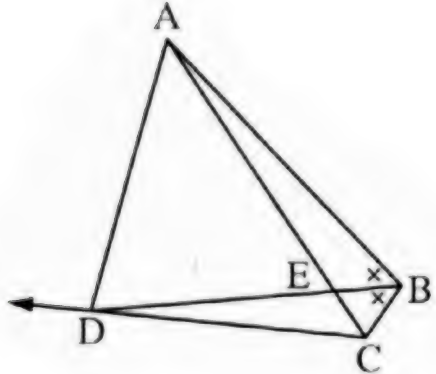
- b) In the opposite figure:  
 M and N are two circles intersecting at A and B.  
 and  $C \in \overrightarrow{BA}$ ,  
 $D \in$  the circle N,  $m(\angle MND) = 130^\circ$ ,  
 $m(\angle BCD) = 50^\circ$ ,  
 Prove that :  $\overrightarrow{CD}$  is a tangent to the circle at D



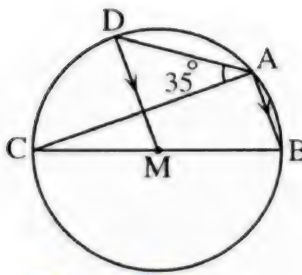
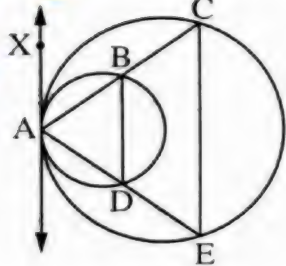
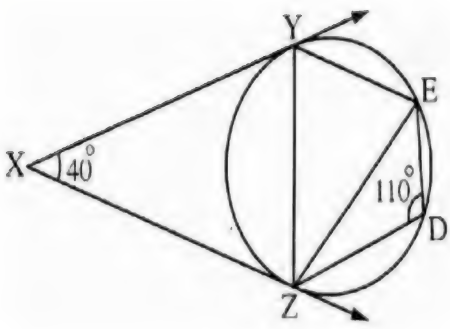
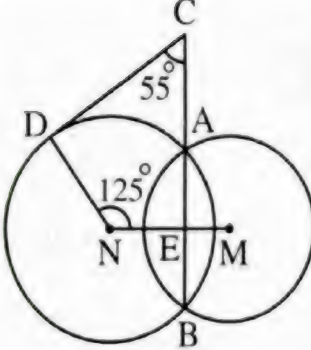


<p>2)</p>	<p>a) In the opposite figure:  <math>\overline{CD}</math> is a tangent to the circle M at C,  <math>\overline{CD} \parallel \overline{BA}</math>  <b>Prove that : <math>m(\angle DCA) = 45^\circ</math></b></p> 
	<p>b) In the opposite figure:  <math>\overline{DE} \parallel \overline{BC}</math>  <b>Prove that</b>  <b><math>m(\angle DAC) = m(\angle BAE)</math></b></p> 
<p>3)</p>	<p>a) In the opposite figure:  <math>\overline{AD} \parallel \overline{BE}</math> , <math>m(\angle BAD) = 100^\circ</math>  <b>And <math>m(\angle CDE) = 30^\circ</math></b>  <b>Find: <math>m(\angle ADC)</math></b></p> 
	<p>b) In the opposite figure  <math>\overline{AB}</math> and <math>\overline{AC}</math> are two tangents  to the circle  M at B and C  <math>\overline{AB} \parallel \overline{CD}</math> ,  <math>m(\angle BMD) = 140^\circ</math>  <b>Find: <math>m(\angle A)</math></b></p> 
<p>4)</p>	<p>a) In the opposite figure:  Two concentric circles at M ,  <math>\overline{AB}</math> and <math>\overline{AC}</math> are two tangent segments to  the smaller circles, <math>m(\angle A) = 70^\circ</math>  <b>(1) Find: <math>m(\angle DME)</math></b>  <b>(2) Prove that : <math>AB = AC</math></b></p> 



	<p>b) In the opposite figure:  <math>\overrightarrow{BC}</math> is a tangent to the circle M at C  D is the midpoint of <math>\overline{EC}</math>, <math>\overline{MC} \parallel \overline{AB}</math>  <b>Prove that : ABCD is a cyclic quadrilateral.</b></p>	
5)	<p>a) In the opposite figure:  A circle of Centre M, <math>\overline{MD} \perp \overline{AB}</math>,  If <math>m(\angle A) = 30^\circ</math>  <b>(1) Prove that : <math>\overline{MD} \parallel \overline{CB}</math></b>  <b>(2) Find : <math>m(\angle C)</math></b></p>	
	<p>b) In the opposite figure:  A circle M, <math>\overline{MD} \perp \overline{AB}</math>,  <math>\overline{ME} \perp \overline{AC}</math> where <math>MD = ME</math>,  <math>m(\angle DME) = 120^\circ</math>  <b>Prove that : the triangle ABC is equilateral.</b></p>	
6)	<p>a) In the opposite figure:  If: <math>AB = AD</math>, <math>m(\angle ABD) = 30^\circ</math>,  <math>m(\angle C) = 60^\circ</math>  <b>Prove that : ABCD is a cyclic quadrilateral</b></p>	
	<p>b) In the opposite figure:  ABCD is a cyclic quadrilateral, <math>\overline{BD}</math>  bisects <math>\angle ABC</math>,  If <math>\overline{BD} \cap \overline{AC} = \{E\}</math>  <b>Prove that : <math>\overrightarrow{CD}</math> is a tangent to the circle</b>  <b>Passing through the vertices of <math>\triangle BEC</math></b></p>	



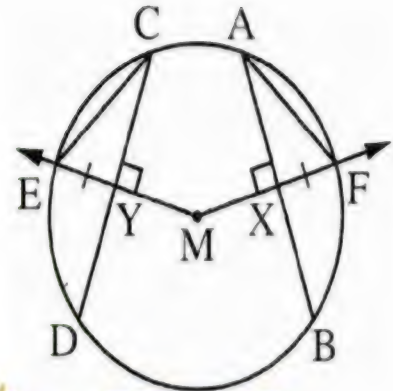
<p>7)</p>	<p>a) In the opposite figure:  <math>\overline{BC}</math> is a diameter in the circle M,  <math>m(\angle CAD) = 35^\circ</math>, <math>\overline{AB} \parallel \overline{DM}</math>,  <b>Find: <math>m(\angle ABC)</math></b></p> 
<p>8)</p>	<p>a) In the opposite figure:              If <math>\overleftrightarrow{AX}</math> is a common tangent to the two circles at A.  <b>Prove that: <math>\overline{BD} \parallel \overline{CE}</math></b></p>  <p>b) In the opposite figure:  <math>\overrightarrow{XY}</math> and <math>\overrightarrow{XZ}</math> are two tangents to the circle from the point X at Y, Z,              if <math>m(\angle EDZ) = 110^\circ</math>, <math>m(\angle YXZ) = 40^\circ</math>  <b>Prove that: <math>m(\widehat{ZDE}) = m(\widehat{ZY})</math></b></p> 
<p>9)</p>	<p>a) In the opposite figure:              M and N are two intersecting circles at A and B,  <math>C \in \overline{BA}</math>, <math>D \in</math> the circle N,  <math>m(\angle MND) = 125^\circ</math> and <math>m(\angle BCD) = 55^\circ</math>  <b>Prove that: <math>\overleftrightarrow{CD}</math> is a tangent to circle N at D</b></p> 



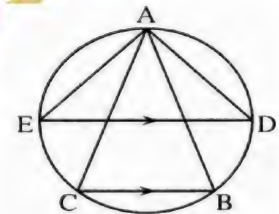
b) In the opposite figure:

$\overline{AB}$  and  $\overline{CD}$  are two chords in the circle M  
 $\overrightarrow{MX} \perp \overline{AB}$  and intersects the circle in F  
 $\overrightarrow{MY} \perp \overline{CD}$  and intersects the circle at E  
 where  $FX = EY$

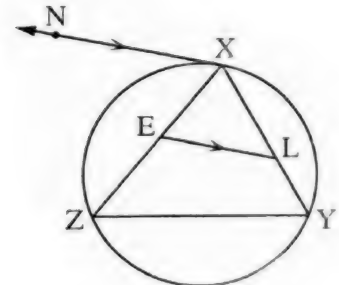
Prove that: (1)  $AB = CD$   
 (2)  $AF = CE$



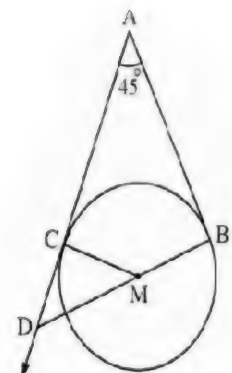
- 10) a) In the opposite figure:  
 ABC is an inscribed triangle inside a circle  
 $\overline{DE} \parallel \overline{BC}$   
 Prove that:  $m(\angle DAC) = m(\angle BAE)$



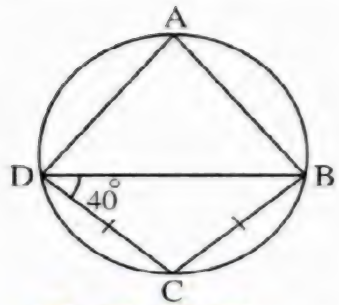
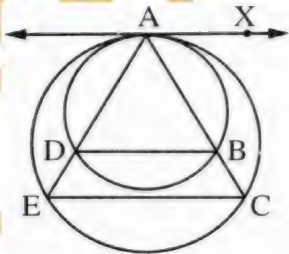
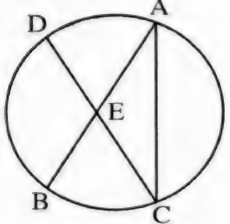
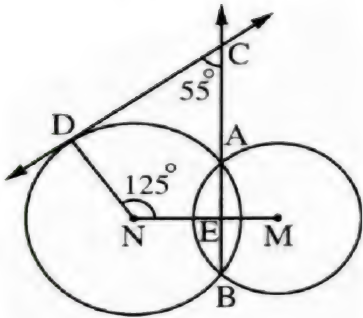
b) In the opposite figure:  
 XYZ is an inscribed triangle in a circle  
 $\overline{LE}$  paralld tangent  $\overrightarrow{XN}$   
 Prove that :  
 LYZE is cyclic quadrilateral.



- 11) a) In the opposite figure:  
 $\overline{AB}$ ,  $\overline{AC}$  are two tangents  
 To circle M at B, C,  
 $m(\angle A) = 45^\circ$   
 Prove that:  
 (1) ABMC is cyclic quadrilateral.  
 (2)  $AD = AB + MB$





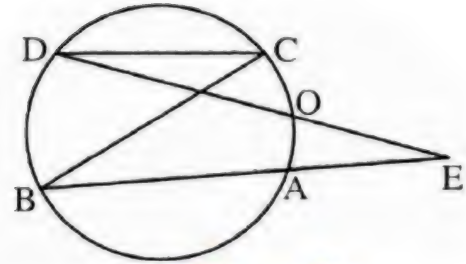
	<p>b) In the opposite figure:  <math>ABCD</math> is a quadrilateral inscribed in circle,  <math>BC = CD</math>, <math>m(\angle BDC) = 40^\circ</math>  <b>Find: <math>m(\angle A)</math></b></p>	
12)	<p>a) In the opposite figure:  <b>Prove that:</b>  <math>\overline{BD} \parallel \overline{CE}</math></p> <p>b) In the opposite figure:  <math>\overline{AB}</math>, <math>\overline{CD}</math> are two equal chords in length  <b>Prove that : the triangle ACE is an isosceles triangle.</b></p>	 
13)	<p>a) In the opposite figure:  <math>M</math> and <math>N</math> are two intersecting circles at <math>A</math> and <math>B</math>, <math>C \in \overrightarrow{BA}</math> and <math>D \in</math> the circle <math>N</math>, <math>m(\angle MND) = 125^\circ</math>, <math>m(\angle BCD) = 55^\circ</math>  <b>Prove that:</b>  <math>\overline{CD}</math> is a tangent to the circle <math>N</math> at <math>D</math></p> <p>b) <math>\overline{AB}</math> and <math>\overline{CD}</math> are two chords in the circle <math>M</math>, <math>\overline{MX}</math> is drawn perpendicular to <math>\overline{AB}</math> to intersect the circle in <math>F</math> and <math>\overline{MY}</math> is drawn perpendicular to <math>\overline{CD}</math> to intersect the circle at <math>E</math>, if <math>FX = EY</math>  <b>Prove that :</b>  <b>(1) <math>AB = CD</math></b>                      <b>(2) <math>AF = CE</math></b></p>	



- 14) a) In the opposite figure:  
E is a point outside the circle

Prove that :

$$m(\angle DCB) > m(\angle E)$$



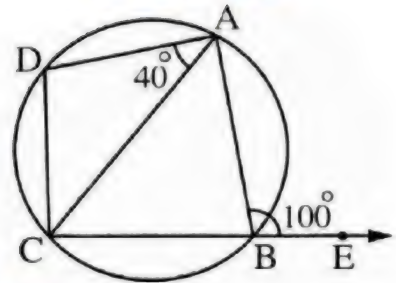
- b) In the opposite figure:

$$m(\angle ABE) = 100^\circ$$

$$, m(\angle CAD) = 40^\circ$$

Prove that:

$$m(\widehat{CD}) = m(\widehat{AD})$$



- 15) Complete:

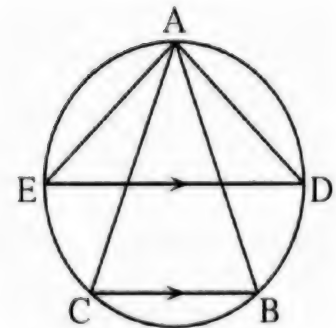
- a) The straight line passing through the center of the circle and the intersection point of the two tangents are ..... to the chord of tangency of those two tangents.

- b) In the opposite figure:

ABC is an inscribed triangle inside the circle

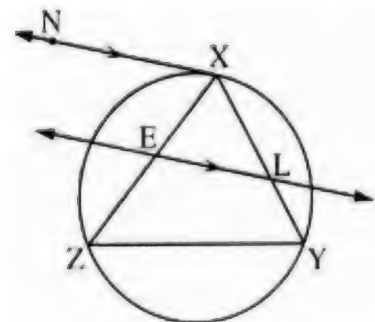
$$, \overline{DE} \parallel \overline{BC}$$

Prove that :  $m(\angle DAC) = m(\angle BAE)$



- 16) In the opposite figure:  
XYZ is an inscribed triangle in a circle , if  $L \in \overline{XY}$  and  $\overline{LE}$  is drawn parallel to the tangent  $\overline{XN}$  which touches the circle at X and intersects  $\overline{XZ}$  at E

Prove that : LYZE is a cyclic quadrilateral.





<p>17)</p>	<p>a) In the opposite figure: A circle M whose radius length is 10 cm., <math>m(\angle DCA) = 40^\circ</math>, <math>AB = 16</math> cm. E is the midpoint of <math>\overline{AB}</math>, <math>\overline{CD}</math> is a tangent to the circle <b>Find by proof: <math>m(\angle DMF)</math>, the length of <math>\overline{FE}</math></b></p>	
<p>18)</p>	<p>a) In the opposite figure: <math>\overline{AB}</math> is a diameter of circle M, <math>m(\angle ABD) = 40^\circ</math> <b>Find <math>m(\angle A)</math>, <math>m(\angle C)</math></b></p>	
<p>19)</p>	<p>b) In the opposite figure: ABC is an inscribed triangle in the circle, <math>\overline{ED} \parallel \overline{BC}</math> <b>Prove that: <math>m(\angle DAC) = m(\angle BAE)</math></b></p>	
<p>20)</p>	<p>a) In the opposite figure: <math>\overline{CB}</math> is a diameter of circle M, <math>\overline{AB} \parallel \overline{DM}</math>, <math>m(\angle DAC) = 30^\circ</math> <b>Find: <math>m(\angle ACB)</math></b></p>	



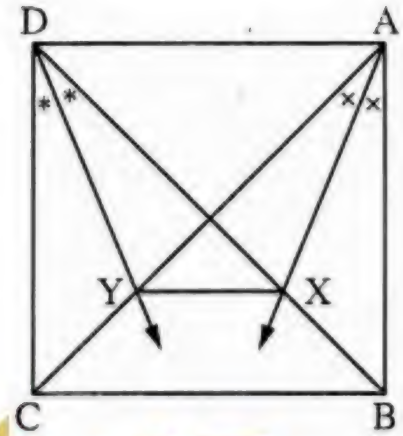
b) In the opposite figure:

ABCD is a square,  $\overrightarrow{AX}$  bisects  $\angle BAC$

and  $\overrightarrow{DY}$  bisects  $\angle CDB$

(1) Prove that the figure AXYD is cyclic quadrilateral

(2) Find with proof  $m(\angle DXY)$



21) In the opposite figure:

$\overrightarrow{XZ}$  and  $\overrightarrow{XY}$  are two tangents at Z and Y,

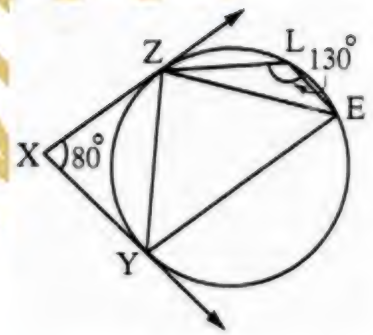
$m(\angle YXZ) = 80^\circ$ ,  $m(\angle ELZ) = 130^\circ$

Prove that:

(1)  $ZE = ZY$

(2)  $\overrightarrow{XZ} \parallel \overrightarrow{YE}$

(3)  $\overrightarrow{ZE}$  is a tangent to the circle passing through the points X, Y and Z



22) a)  $\overline{AB}$  is a diameter in the circle M,  $\overline{AC}$  is a chord in it where  $m(\angle BAC) = 30^\circ$ ,  $\overline{BC}$  is drawn and  $\overline{MD}$  is drawn perpendicular to  $\overline{AC}$  and intersect it in D,

Prove that

(1)  $\overline{MD} \parallel \overline{BC}$

(2) The length of  $\overline{BC}$  = length of radius.

b) In the opposite figure:

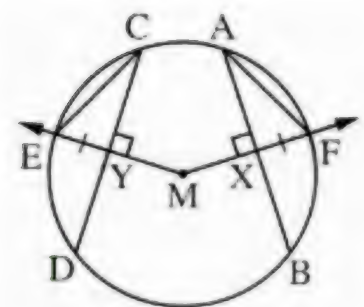
$\overline{AB}$  and  $\overline{CD}$  are two chords in the circle M,

$\overrightarrow{MX} \perp \overline{AB}$  and intersect it at F

,  $\overrightarrow{MY} \perp \overline{CD}$  and intersect it at E

,  $FX = EY$

Prove that: (1)  $AB = CD$  (2)  $AF = CE$





23) a) Prove that:

In a cyclic quadrilateral each two opposite angles are supplementary.

b) In the opposite figure:

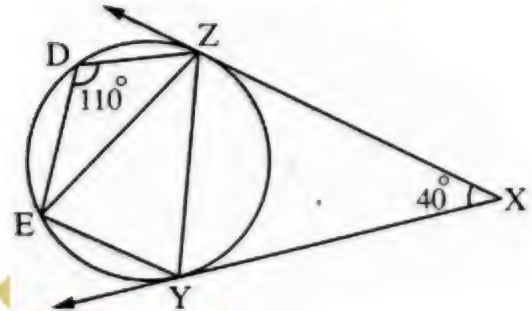
$\overrightarrow{XY}, \overrightarrow{XZ}$  are two tangents to the circle from point X,

$m(\angle D) = 110^\circ$ ,

$m(\angle X) = 40^\circ$

Prove that

$m(\widehat{ZE}) = m(\widehat{ZY})$



24) a) A is a point outside a circle M,  $\overrightarrow{AB}$  is a tangent to the circle at point B,  $\overrightarrow{AM}$  intersects the circle M at C and D respectively,  $m(\angle A) = 40^\circ$  and draw  $\overline{BM}$

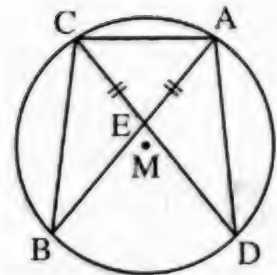
Find with proof:  $m(\angle BDC)$

b) In the opposite figure

$\overline{AB}, \overline{CD}$  are two chords in the circle

M intersecting at E, If  $AE = CE$

Prove that:  $m(\angle ACB) = m(\angle CAD)$



25)  $\overline{AB}$  is a diameter in the circle M,  $\overline{AC}$  is a chord in this circle and D is the midpoint of  $\overline{AC}$ ,  $\overrightarrow{DM}$  was drawn to intersect the tangent to the circle M at B in E

Prove that:

(1) The figure ADBE is cyclic quadrilateral.

(2)  $m(\angle CMB) = 2m(\angle MEB)$



۱۲) اجتماع

$\therefore \overrightarrow{BD}$  is a tangent to the circle at B

tangency and inscribed angles subtended by  $\widehat{AB}$

from ① and ②  $\therefore m(\angle BX\gamma) = m(\angle C)$

$\therefore$   $AXYC$  is a cyclic quadrilateral.

$\vec{AC}$  is a tangent to the smaller circle at C

$$AC = 15 \text{ cm}, AB = (2x - 3) \text{ cm}$$

Solution:-

$\therefore \overline{AB}$  and  $\overline{AD}$  are two tangents to the greater circle  $\therefore AB = AD \Rightarrow y - 2 = 15 \Rightarrow \boxed{y = 17}$

Circle  $\therefore AB = AD \Rightarrow y - 2 = 15 \Rightarrow \boxed{y = 17}$



In the opposite figure:

$\overline{AB}$  is a diameter in the circle  $M$

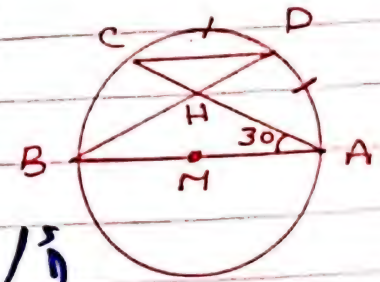
,  $C \in$  the circle  $M$ ,  $m(\angle CAB) = 30^\circ$

,  $D$  is midpoint of  $\widehat{AC}$ ,  $\overline{DB} \cap \overline{AC} = \{H\}$

IV) Find:  $m(\angle BDC)$  and  $m(\widehat{AD})$

V) prove that:  $\overline{AB} \parallel \overline{DC}$

والمعروف



Solution:

$$m(\angle BDC) = m(\angle BAC) = 30^\circ$$

(first)

two inscribed angles subtended by  $\widehat{BC}$

$$\therefore m(\widehat{BC}) = 60^\circ$$

$$\therefore \overline{AB} \text{ is a diameter } \therefore m(\widehat{AB}) = 180^\circ$$

$\therefore D$  is midpoint of  $\widehat{AC}$

$$\therefore m(\widehat{AD}) = m(\widehat{DC}) = \frac{180 - 60}{2} = \frac{120}{2} = 60^\circ$$

$$\therefore m(\angle DCA) = \frac{1}{2} m(\widehat{AD}) = 30^\circ$$

$\therefore m(\angle BAC) = m(\angle DCA)$  and they are alternate

$$\therefore \overline{AB} \parallel \overline{DC}$$

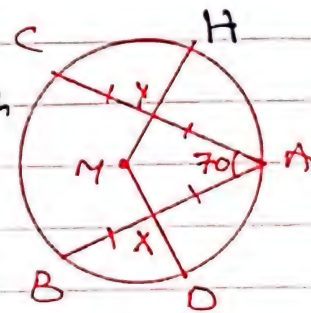
In the opposite figure:

$\overline{AB}$  and  $\overline{AC}$  are two equal chords in length

in circle  $M$ ,  $x$  is the midpoint of  $\overline{AB}$ ,  $y$

is the midpoint of  $\overline{AC}$ ,  $m(\angle CAB) = 70^\circ$

VI) Calculate:  $m(\angle DMH)$  VII) prove that:  $XD = YH$



Solution:

$$\therefore x \text{ is the midpoint of } \overline{AB} \therefore \overline{MX} \perp \overline{AB} \dots \textcircled{1}$$

$$\therefore y \text{ is the midpoint of } \overline{AC} \therefore \overline{MY} \perp \overline{AC} \dots \textcircled{2}$$

$$\therefore AB = AC \dots \textcircled{3}$$

$\therefore$  the sum of interior angles of quadrilateral =  $360^\circ$

$$\therefore m(\angle DMH) = 360 - (90 + 90 + 70) = 110^\circ$$

$$\text{from } \textcircled{1}, \textcircled{2} \text{ and } \textcircled{3} \quad MX = MY \dots \textcircled{4} \therefore MD = MH = r \dots \textcircled{5}$$

$$\text{by subtracting } \textcircled{4} \text{ from } \textcircled{5} \therefore XD = YH$$



In the opposite figure:

$$m(\angle A) = 30^\circ, m(\widehat{AC}) = 120^\circ.$$

$$m(\widehat{BC}) = m(\widehat{DH})$$

- ① find:  $m(\widehat{BD})$  the minor

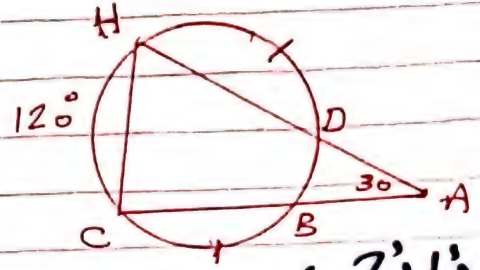
- ② prove that:  $AB = AD$

Solution:

$$m(\angle A) = \frac{1}{2} [m(\widehat{HC}) - m(\widehat{DB})]$$

$$30 = \frac{1}{2} [120 - m(\widehat{BD})]$$

$$60 = 120 - m(\widehat{BD}) \Rightarrow \boxed{m(\widehat{BD}) = 60^\circ}$$



ذا الذرعر

$$\therefore m(\widehat{HD}) = m(\widehat{CB}) \therefore HD = CB \rightarrow \textcircled{D}$$

$$c_{\text{ind}} m(\widehat{HD}) + m(\widehat{DB}) = m(\widehat{CB}) + m(\widehat{DB})$$

$$\therefore m(\widehat{HDB}) = m(\widehat{CDB}) \Rightarrow m(\angle C) = m(\angle H)$$

$$\therefore AH \cong AC \rightarrow \textcircled{2}$$

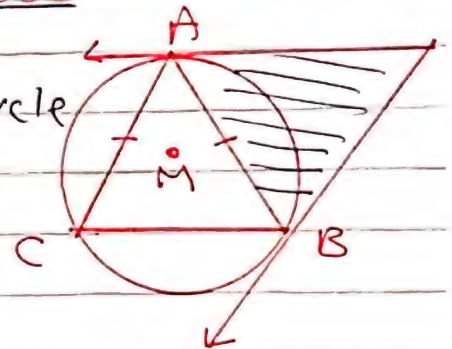
by subtracting ① from ②  $\therefore AB = AD$

In the opposite figure:

$\vec{DA}$  and  $\vec{DB}$  are Two Tangents of the Circle

M and  $AB=AC$  prove that:

$\vec{AC}$  is a tangent to the circle passing through the vertices of  $\triangle ABD$ .



Solution:

# hint

we need to prove that  $m(\angle BAC) = m(\angle$

Solution  $\therefore \vec{DA}$  is tangent to the circle at A

$$\therefore m(\angle DAB) \text{ tangency} = m(\angle ACB) \text{ inscribed} \quad \dots (i)$$

$$\therefore AC = AB \rightarrow m(\angle ACB) = m(\angle ABC) \rightarrow (2)$$

$\therefore \vec{DA}$  and  $\vec{DB}$  are two tangents  $\therefore DA = DB$

$$\therefore m(\angle DAB) = m(\angle DBA) \quad \dots (3)$$

from ① and ② and ③  $\therefore m(\angle CAB) = m(\angle D)$

$\therefore \vec{AC}$  is a tangent  $\therefore$



(4)

In the opposite figure:

$C$  is the midpoint of  $\overline{AB}$ ,  
 $\overrightarrow{MC} \cap$  the Circle  $M = \{D\}$ ,  
 $m(\angle MAB) = 20^\circ$

Find:  $m(\angle BHD)$  and  $m(\widehat{ADB})$

Solution:

$$\therefore MA = MB = r$$

$$\therefore m(\angle A) = m(\angle MBA) = 20^\circ$$

$\therefore C$  is the midpoint of  $\overline{AB}$   $\therefore \overline{MC} \perp \overline{AB}$

In  $\triangle MBC$

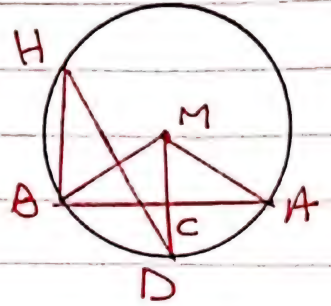
$$m(\angle BMC) = 180 - (90 + 20) = 70^\circ$$

$$\Rightarrow \therefore m(\angle AMB) = \frac{1}{2} m(\angle BMD) = 35^\circ$$

inscribed angle and central angle subtended by  $\widehat{BD}$

$$m(\angle AMB) = 180 - (20 + 20) = 140^\circ$$

$$\therefore m(\widehat{ADB}) = m(\angle AMB) = 140^\circ$$



In the opposite figure:

$AB = AC$ ,  $\overline{MD} \perp \overline{AB}$ ,  $\overline{ME} \perp \overline{AC}$

prove that:  $XD = YE$

Solution

$$\therefore \overline{MD} \perp \overline{AB}$$

$$\text{, } \overline{ME} \perp \overline{AC}$$

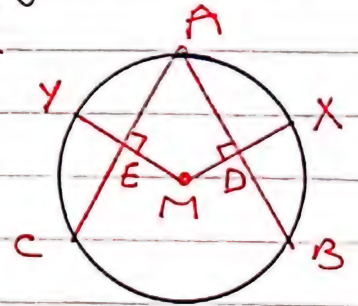
$$\text{and } AB = AC$$

$$\therefore ME = MD \text{ --- (1)}$$

$$\therefore MY = MX = r \text{ --- (2)}$$

by subtracting (1) from (2)

$$\therefore YE = XD \quad \#$$



أنا أفرح



In the opposite figure:

ABCD is a quadrilateral in which

$$AB = AD, m\angle ABD = 30^\circ, m\angle C = 60^\circ$$

prove that: ABCD is a cyclic quad.

Solution:

In  $\triangle ABD$   $\because AB = AD$

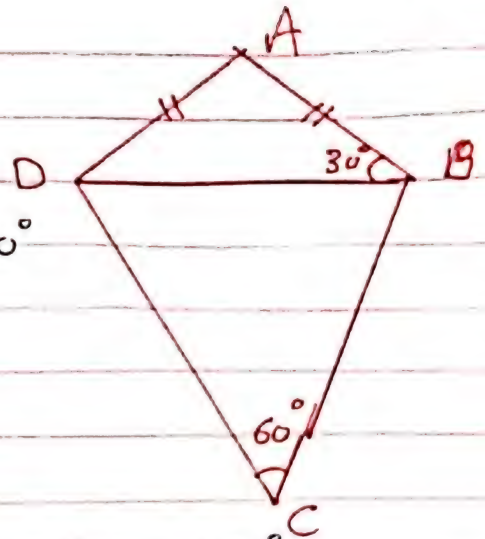
$$\therefore m\angle ABD = m\angle ADB = 30^\circ$$

$$\therefore m\angle DAB = 180 - (30 + 30) = 180 - 60 = 120^\circ$$

$$\therefore m\angle A + m\angle C = 60 + 120 = 180$$

and they are opposite angle

$\therefore$  ABCD is a cyclic quadrilateral.



المرح

In the opposite figure:

$\widehat{BC}$  is a tangent at B;

E is the midpoint of  $\widehat{BF}$

prove that: ABCD is a cyclic quad.

Solution:

$\therefore \widehat{BC}$  is a tangent at B

$$\therefore m\angle CBE \text{ tangency} = m\angle BAE$$

inscribed --- ①

$\therefore$  E is the midpoint of  $\widehat{BF}$

$$\therefore m(\widehat{BE}) = m(\widehat{EF})$$

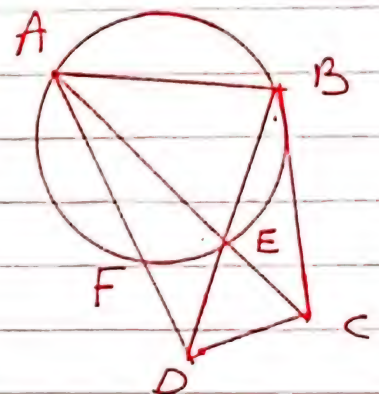
$$\therefore m\angle BAE = m\angle EAF \text{ --- ②}$$

inscribed angles subtended by equal arcs.

from ① and ②  $\therefore m\angle CBD = m\angle CAD$

drawn on  $\widehat{CD}$  and on one side of it

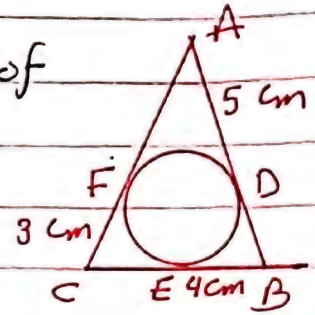
$\therefore$  ABCD is a cyclic quadrilateral.





In the opposite figure:

A circle is drawn touches the sides of a triangle  $ABC$ ,  $\overline{AB}$ ,  $\overline{BC}$ ,  $\overline{AC}$  at  $D$ ,  $E$ ,  $F$ ,  $AD = 5 \text{ cm}$ ,  $BE = 4 \text{ cm}$ ,  $CF = 3 \text{ cm}$



Find: The perimeter of  $\triangle ABC$

Solution:

$\therefore \overline{AD}, \overline{AF}$  are two tangent-segments

$$\therefore AD = AF = 5 \text{ cm}$$

$\therefore \overline{BD}, \overline{BE}$  are two tangent-segments

$$\therefore BD = BE = 4 \text{ cm}$$

and  $\therefore \overline{CF}, \overline{CE}$  are two tangent-segments

$$\therefore CE = CF = 3 \text{ cm}$$

$$\therefore \text{Perimeter of } \triangle ABC = 5 + 5 + 4 + 4 + 3 + 3 = 24 \text{ cm}$$

In the opposite figures

$\overline{AB}, \overline{AC}$  are two tangents to the circle at  $B, C$ ,  $m(\angle D) = 125^\circ$   
 $m(\angle A) = 70^\circ$  prove that:

①  $CB = CE$

②  $\overline{AC} \parallel \overline{BE}$

Solution

$\therefore \overline{AB}$  and  $\overline{AC}$  tangent-segments

$$\therefore AB = AC$$

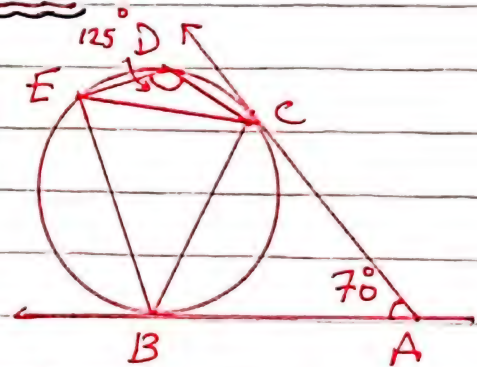
$$\therefore m(\angle ACB) = m(\angle ABC) = \frac{180 - 70}{2} = 55^\circ$$

$$\therefore m(\angle ACB) \text{ tangency} = m(\angle BEC) = 55^\circ$$

inscribed

$\therefore EBCD$  is cyclic quad

$$\therefore m(\angle B) = 180 - 125 = 55^\circ$$



In  $\triangle EBC$

$$\therefore m(\angle EBC) = m(\angle ECB)$$

$$\therefore EC = BC \text{ (first)}$$

$$\therefore m(\angle ACB) = m(\angle CBE)$$

and they are alternate

$$\therefore \overline{AC} \parallel \overline{BE}$$



In the opposite figure:

$\overline{BC}$  is a diameter in the Circle M

$\overline{ED} \perp \overline{BC}$

prove that: ① ABDE is a cyclic quad.

②  $m(\angle DEC) = \frac{1}{2} m(\widehat{AC})$

Solution:

$\because \overline{BC}$  is a diameter  $\therefore m(\angle BAC) = 90^\circ$

inscribed drawn in a semicircle

$\because \overline{ED} \perp \overline{BC} \therefore m(\angle EDB) = 90^\circ$

$\therefore m(\angle BAE) + m(\angle EBD) = 90 + 90 = 180^\circ$

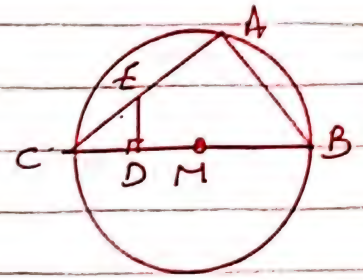
and they are opposite

$\therefore$  ABDE is a cyclic quadrilateral

$\therefore m(\angle DEC)_{\text{exterior}} = m(\angle B)_{\text{interior}}$

and  $\therefore m(\angle B) = \frac{1}{2} m(\widehat{AC})$

$\therefore m(\angle DEC) = \frac{1}{2} m(\widehat{AC})$

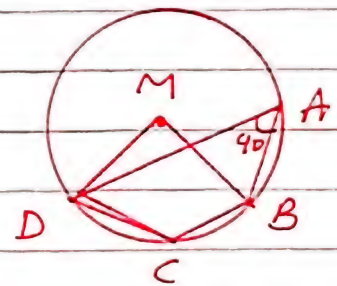


In the opposite figure

Find ①  $m(\angle BMD)$  ②  $m(\angle BCD)$

Solution:

$m(\angle BMD)_{\text{central}} = 2m(\angle BAD)_{\text{inscribed}} = 80^\circ$



$m(\angle BCD)_{\text{inscribed}} = \frac{1}{2} (\text{reflex } \angle BMD)_{\text{central}} = \frac{1}{2} \times 280 = 140^\circ$

another solution:

$\therefore$  ABCD is a cyclic quadrilateral

$\therefore m(\angle C) = 180 - 40 = 140^\circ$



8

In the opposite figure:

D is the midpoint of  $\overline{AB}$ ,  $\overline{ME} \perp \overline{AC}$

$m(\angle DME) = 140^\circ$  and  $m(\angle C) = 70^\circ$

prove that:  $MD = ME$

Solution:

$\therefore$  D is the midpoint of  $\overline{AB}$

$\therefore \overline{MD} \perp \overline{AB}$

$\therefore$  the Sum of interior angles of the quad. =  $360^\circ$

$\therefore m(\angle A) = 360 - (90 + 90 + 140) = 40^\circ$

In  $\triangle ABC$

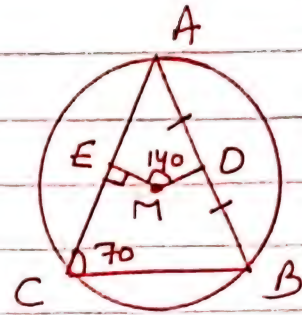
$m(\angle B) = 180 - (70 + 40) = 70^\circ$

$\therefore m(\angle B) = m(\angle C)$

$\therefore AB = AC$

$\therefore \overline{MD} \perp \overline{AB}$ ,  $\overline{ME} \perp \overline{AC}$  and  $AB = AC$

$\therefore MD = ME$



In the opposite figure:

find each of

(1)  $m(\widehat{AD})$

(2)  $m(\angle AXD)$

Solution

$m(\widehat{BC}) = 2m(\angle A) = 60^\circ$

$\therefore \overline{AC} \cap \overline{DB} = \{E\}$

$\therefore m(\angle E) = \frac{1}{2} [m(\widehat{AD}) - m(\widehat{BC})]$

$\therefore 50 = \frac{1}{2} [m(\widehat{AD}) - 60]$

$\times 2$

$100 = m(\widehat{AD}) - 60$

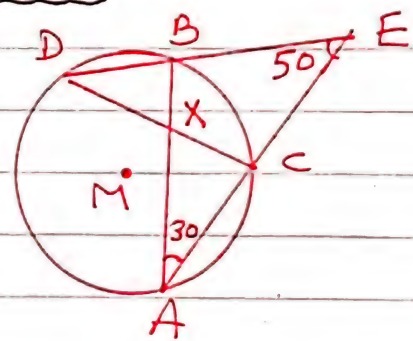
$\Rightarrow m(\widehat{AD}) = 100 + 60 = 160^\circ$  "first"

$\therefore \overline{DC} \cap \overline{AB} = \{X\}$

$\therefore m(\angle AXD) = \frac{1}{2} [m(\widehat{AD}) + m(\widehat{BC})]$

$= \frac{1}{2} [160 + 60] = 110^\circ$

"second"





9

In the opposite figure:

$\overrightarrow{AB}$  is a tangent to the circle M

E is midpoint of  $\overline{XY}$

$$m(\angle BME) = 130^\circ$$

Find  $m(\angle A)$

Solution:

$\therefore \overrightarrow{AB}$  is a tangent  $\therefore \overline{MB} \perp \overrightarrow{AB}$

$\therefore E$  is the midpoint of  $\overline{XY} \therefore \overline{ME} \perp \overline{XY}$

$\therefore$  the sum of interior angles of quad. =  $360^\circ$

$$\therefore m(\angle A) = 360 - (130 + 90 + 90) = 50^\circ$$

إلى آخره

$\Rightarrow$

In the opposite figure:

$$m(\angle ABC) = 110^\circ$$

find  $m(\widehat{ABC})$

Solution:

$$m(\widehat{AC}) = 2m(\angle ABC) = 220^\circ$$

$\therefore$  the measure of the circle =  $360^\circ$

$$\therefore m(\widehat{ABC}) = 360 - 220 = 140^\circ$$

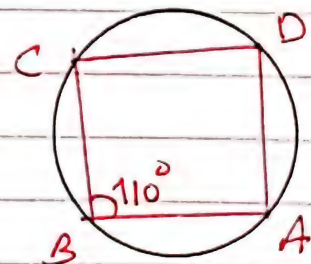
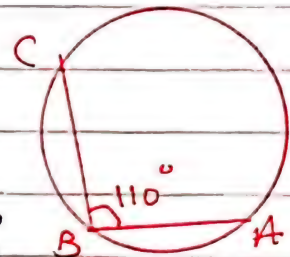
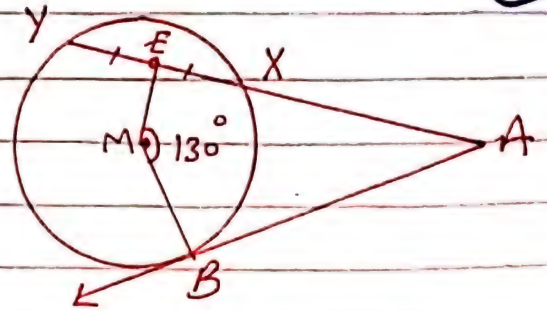
another solution:

put D on the circle and draw  $\overline{DA}$  and  $\overline{DC}$

$\therefore ABCD$  is a cyclic quad.

$$\therefore m(\angle D) = 180 - 110 = 70^\circ$$

$$\therefore m(\widehat{ABC}) = 2m(\angle D) = 140^\circ$$





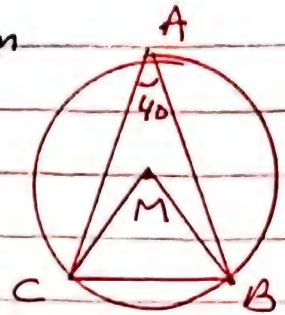
M is a circle with radius length 6.3 cm

$$m(\angle BAC) = 40^\circ$$

Find (1)  $m(\angle BMC)$

(2) length  $(\widehat{BC})$  where  $(\pi \approx \frac{22}{7})$

Solution:



$$m(\angle BMC) = 2 m(\angle BAC) = 80^\circ$$

inscribed angle and Central angle subtended by  $\widehat{BC}$

In  $\triangle MBC$

$$\therefore MB = MC = r$$

$$\therefore m(\angle MBC) = m(\angle MCB) = \frac{180 - 80}{2} = 50^\circ$$

$$\text{length}(\widehat{BC}) = \frac{m(\widehat{BC})}{360} \times 2\pi r$$

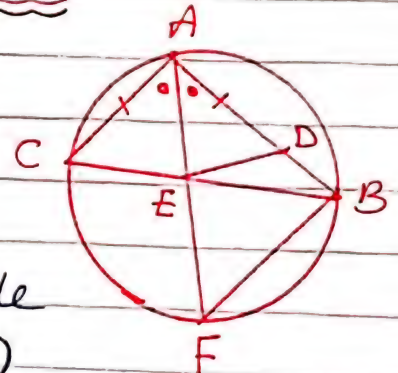
المساحة / د

$$= \frac{80}{360} \times 2 \times \frac{22}{7} \times 6.3 = 8.8 \text{ cm}$$

$AD = AC$ ,  $\vec{AF}$  bisects  $\angle A$

prove that: DBFE is a cyclic quad.

Solution:



In  $\triangle ADE, ACE$   $\begin{cases} AD = AC \\ \vec{AE} \text{ is a common side} \\ m(\angle DAE) = m(\angle CAE) \end{cases}$

$$\therefore \triangle ADE \equiv \triangle ACE$$

$$\therefore m(\angle ADE) = m(\angle ACE) \quad \dots \textcircled{1}$$

$$\therefore m(\angle ACB) = m(\angle AFB) \quad \dots \textcircled{2}$$

two inscribed angles subtended by  $\widehat{AB}$   
from  $\textcircled{1}$  and  $\textcircled{2}$

$$\therefore m(\angle ADE) \text{ exterior} = m(\angle EFB) \text{ interior}$$

$\therefore$  DBFE is a cyclic quad.

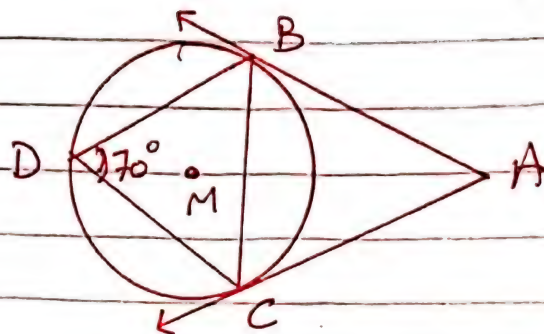


$\vec{AB}, \vec{AC}$  are two tangents to the circle  $M$ ,  $m\angle BDC = 70^\circ$

Find:  $m\angle A$

Solution:

$\therefore \vec{AB}$  is a tangent



$\therefore m\angle ABC$  tangency =  $m\angle D$  inscribed =  $70^\circ$   
subtended by the arc  $\widehat{BC}$

$\therefore \vec{AB}, \vec{AC}$  are two tangent-segments

$\therefore AB = AC$

$\therefore m\angle ABC = m\angle ACB = 70^\circ$

$\therefore m\angle A = 180 - (70 + 70) = 40^\circ$

المعلم / P

$\vec{AB}$  is a diameter in the circle  $M$   
 $\vec{CD}$  is a tangent to the circle at  $C$   
 $\vec{CD} \parallel \vec{AB}$

(1) prove that:  $AC = BC$

(2) find:  $m\angle B$  by degree

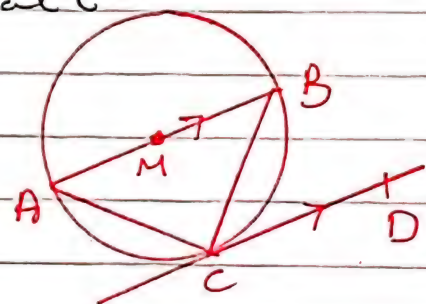
Solution:

$\therefore \vec{CD}$  is a tangent  
and  $\vec{CD} \parallel \vec{AB}$

$\therefore m(\widehat{BC}) = m(\widehat{AC})$

$\therefore BC = AC$

$\rightarrow$  ①  
first



$\therefore \vec{AB}$  is a diameter

$m(\widehat{BC}) + m(\widehat{AC}) = 180 \rightarrow$  ②

from ① and ②  $\therefore m(\widehat{AC}) = 90^\circ$

$\therefore m\angle B$  inscribed =  $\frac{1}{2} m(\widehat{AC}) = 45^\circ$



(12)

$\vec{AB}$  is a tangent to the circle  $M$  at  $B$ ,  $m(\angle A) = 40^\circ$

find:  $m(\angle BDC)$

Solution:

$\therefore \vec{AB}$  is a tangent at  $B$   
 $\therefore \vec{MB} \perp \vec{AB}$

In  $\triangle AMB$ :  $m(\angle AMB) = 180 - (90 + 40) = 50^\circ$

$\therefore m(\angle BDC) \text{ tangency} = \frac{1}{2} m(\angle BMC) \text{ central}$   
 $= 25^\circ$

أو نصف القوس

Find:  $m(\widehat{CD})$

Solution

$m(\angle H) = \frac{1}{2} [m(\widehat{AC}) - m(\widehat{DB})]$

$\therefore 30 = \frac{1}{2} [80 - m(\widehat{DB})] \quad (\times 2)$

$60 = 80 - m(\widehat{DB}) \Rightarrow m(\widehat{DB}) = 80 - 60 = 20^\circ$

$\therefore \vec{AB}$  is a diameter

$\therefore m(\widehat{AC}) + m(\widehat{DC}) + m(\widehat{DB}) = 180^\circ$

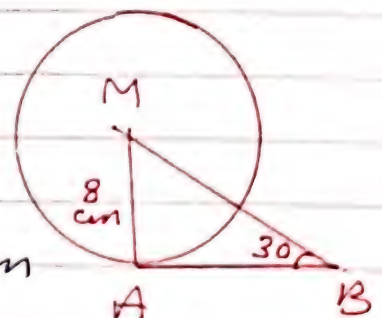
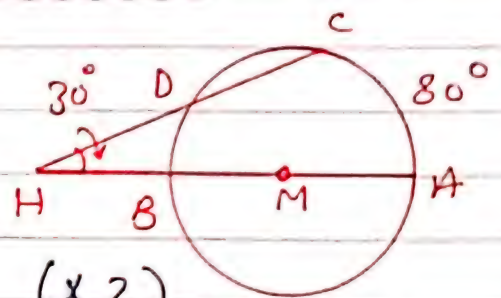
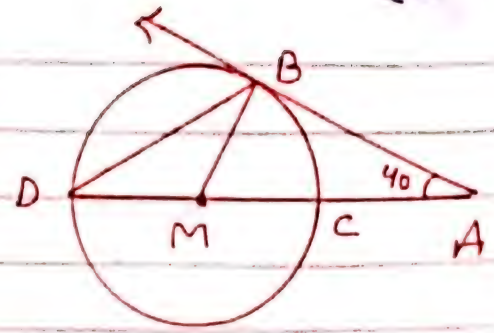
$\therefore m(\widehat{DC}) = 180 - (80 + 20) = 80^\circ$

Find: length of  $\vec{AB}$

Solution:

$\therefore \vec{AB}$  is a tangent  $\therefore \vec{MA} \perp \vec{AB}$

$\therefore \tan 30 = \frac{8}{AB} \Rightarrow AB = \frac{8}{\tan 30} = 8\sqrt{3} \text{ cm}$



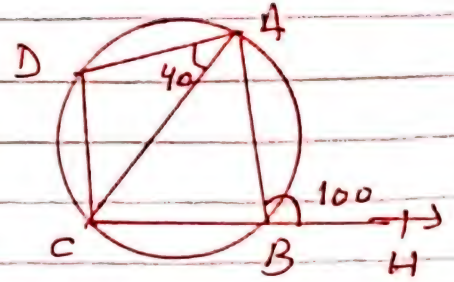


prove that:  $AD = DC$

Solution:

$\therefore ABCD$  is a cyclic quad.

$$\therefore m(\angle ADC) \text{ interior} = m(\angle ABH) \\ = 100^\circ \text{ exterior}$$



In  $\triangle ADC$ :

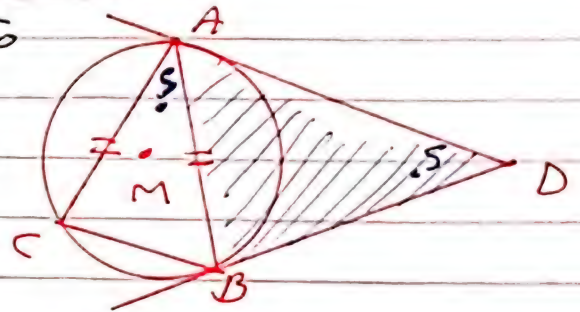
$$m(\angle ACD) = 180 - (100 + 40) = 40^\circ$$

$$\therefore m(\angle DAC) = m(\angle DCA) = 40^\circ$$

$$\therefore DC = DA$$

مربع

$\vec{DA}$  and  $\vec{DB}$  are two tangents to the circle  $M$  and  $AB = AC$   
 prove that:  $\vec{AC}$  is a tangent to the circle passing through the vertices of  $\triangle ABD$



Hint we need to prove that  $m(\angle D) = m(\angle CAB)$

Solution:

In  $\triangle ABD$

$$\therefore AB = AC \therefore m(\angle ACB) = m(\angle ABC) \dots \textcircled{1}$$

In  $\triangle DAB$

$\therefore \vec{DA}, \vec{DB}$  are two tangent segments

$$\therefore DA = DB$$

$$\therefore m(\angle DBA) = m(\angle DAB) \dots \textcircled{2}$$

$\therefore \vec{DA}$  is a tangent

$$\therefore m(\angle DAB) \text{ tangency} = m(\angle ACB) \text{ inscribed} \dots \textcircled{3}$$

from  $\textcircled{1}, \textcircled{2}$  and  $\textcircled{3} \therefore m(\angle CAB) = m(\angle D)$   
 $\therefore \vec{AC}$  is a tangent to the circle passing through the vertices of  $\triangle ABD$



$\vec{AB}, \vec{AC}$  are two tangents to the circle at B, C

prove that:  $\vec{BC}$  bisects  $\angle ABH$

Solution:

$\therefore \vec{AB}, \vec{AC}$  are two tangent-segments

$$\therefore AB = AC$$

$$\therefore m(\angle ABC) = m(\angle ACB) = \frac{180 - 50}{2} = \frac{130}{2} = 65^\circ \quad \dots (1)$$

$\therefore DCBH$  is a cyclic quadrilateral

$$\therefore m(\angle HBC) = 180 - 115 = 65^\circ \quad \dots (2)$$

from (1) and (2)

$$\therefore m(\angle ABC) = m(\angle HBC)$$

$\therefore \vec{BC}$  bisects  $\angle ABH$

ABC is a triangle inscribed in a circle,  $X \in \widehat{AB}$ ,  $Y \in \widehat{AC}$ , where  $m(\widehat{AX}) = m(\widehat{AY})$ ,  $\overline{CX} \cap \overline{AB} = \{D\}$ ,  $\overline{BY} \cap \overline{AC} = \{H\}$

prove that: BCHD is a cyclic quadrilateral.

Solution

$$\therefore m(\widehat{AX}) = m(\widehat{AY})$$

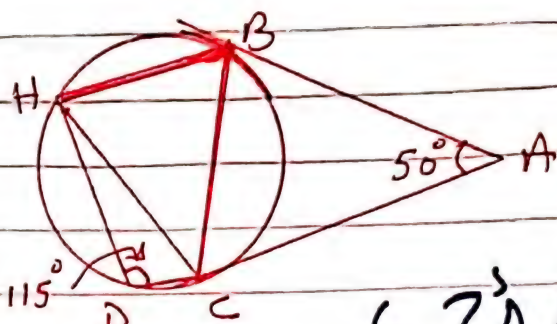
$$\therefore m(\angle ACX) = m(\angle ABY)$$

inscribed angles subtended by equal arcs.

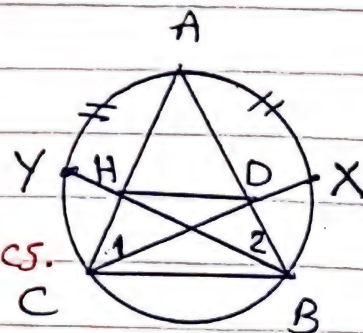
In the quad. BCHD

$$\therefore m(\angle DBH) = m(\angle DCH)$$

drawn on DH and on one side of it  
= BCHD a cyclic quad.



المسألة 14





$\overline{AB}$  and  $\overline{AC}$  are two tangents segments to the circle

$M, m(\angle BAM) = 25$

find: ①  $m(\angle ACB)$

②  $m(\angle BEC)$

Solution:

$\therefore \overline{AB}$  and  $\overline{AC}$  are two tangents

$\therefore \overleftrightarrow{AM}$  bisects  $\angle A$  and  $\angle CMB$   
and  $AC = AB$

$\therefore m(\angle A) = 2 \times 25 = 50^\circ$

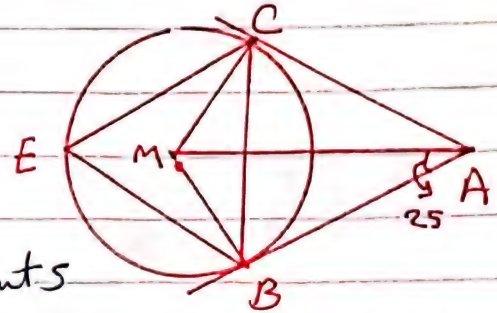
$m(\angle ACB) = (180 - 50) \div 2 = 65^\circ$  (first)

and  $\overline{MC} \perp \overline{AC}, \overline{MB} \perp \overline{AB}$

$\therefore ABMC$  is a cyclic quadrilateral

$\therefore m(\angle M) + m(\angle A) = 180 \Rightarrow m(\angle CMB) = 180 - 50 = 130^\circ$

$\therefore m(\angle BEC) \text{ inscribed} = \frac{1}{2} m(\angle CMB) \text{ central} = 65^\circ$



25/P

$ABED$  is a parallelogram in which  $AC = AB$

prove that:  $\overleftrightarrow{EB}$  is a tangent to the circle circumscribed about the triangle  $ABC$ .

Solution:

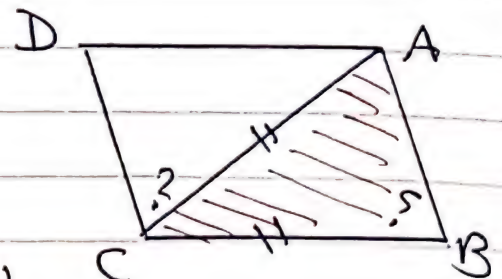
Hint: we need to prove that

$m(\angle DCA) = m(\angle B)$

Solution:

$\therefore ABED$  is a parallelogram

$\therefore m(\angle DCA) = m(\angle CAB)$  "alt." ... ①



In  $\triangle ABC$

$\therefore AC = CB \therefore m(\angle ABC) = m(\angle CAB)$  ... ②

from ① and ②  $\therefore m(\angle DCA) = m(\angle ABC)$

$\therefore \overleftrightarrow{EB}$  is a tangent to the circle passes through the vertices of  $\triangle ABC$







prove that:  $\vec{AD}$  is a tangent to the circle which passes through the vertices of  $\triangle ABC$

Solution:

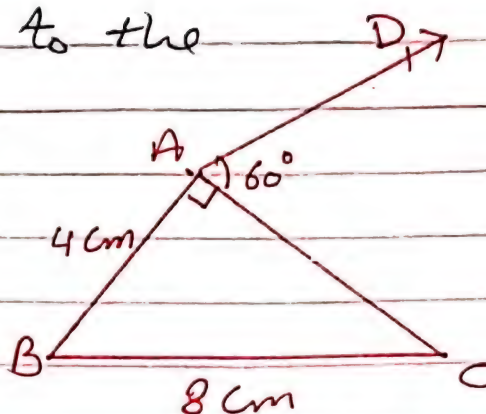
In  $\triangle ABC$

$$\cos(B) = \frac{4}{8} = \frac{1}{2}$$

$$\therefore m(\angle B) = 60^\circ$$

$$\therefore m(\angle DAC) = m(\angle B) = 60^\circ$$

$\therefore \vec{AD}$  is a tangent to the circle which passes through the vertices of  $\triangle ABC$



المسألة / 9

$m(\widehat{AEC}) = 100^\circ$ ,  $\overline{BC}$  is a diameter

$$m(\angle D) = 30^\circ$$

find:  $m(\angle BAD)$

Solution:

$$\therefore m(\angle D) = \frac{1}{2} [m(\widehat{AB}) - m(\widehat{EC})]$$

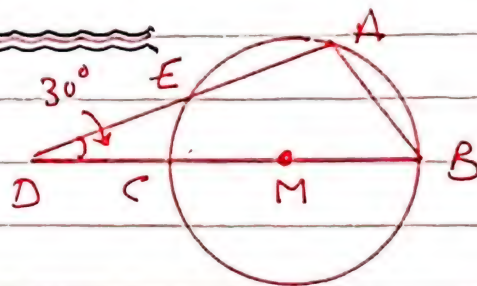
$$\because \overline{AB} \text{ is a diameter } \therefore m(\widehat{BAC}) = 180^\circ$$

$$\therefore m(\widehat{AB}) = 180 - 100 = 80^\circ$$

$$\therefore 30 = \frac{1}{2} [80 - m(\widehat{EC})] \quad (\times 2)$$

$$80 - m(\widehat{EC}) = 60 \Rightarrow m(\widehat{EC}) = 20^\circ$$

$$\therefore m(\angle BAD) = m(\angle BAE) = \frac{1}{2} m(\widehat{BCE}) = \frac{1}{2} \times 200 = 100^\circ$$



$\overline{CM} \parallel \overline{AB}$  prove that:  $BE > AE$

Solution:  $\therefore m(\angle CMA) = 2m(\angle CBA)$

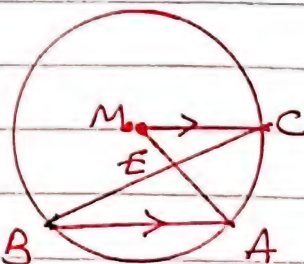
$$\therefore m(\angle CMA) > m(\angle CBA) \quad \text{--- ①}$$

$$\because \overline{MC} \parallel \overline{BA} \therefore m(\angle CMA) = m(\angle MAB) \quad \text{--- ②}$$

$$\text{from ① and ② } \therefore m(\angle MAB) > m(\angle CBA)$$

$$\therefore \triangle EAB \therefore m(\angle EAB) > m(\angle EBA)$$

$$\therefore EB > EA$$





length of  $(\widehat{BX}) = \text{length of } (\widehat{CY})$

prove that:  $AX = AY$

Solution:

$\therefore \text{length of } (\widehat{BX}) = \text{length of } (\widehat{CY})$

$\therefore m(\widehat{BX}) = m(\widehat{CY})$

by adding  $m(\widehat{XY})$  to the two sides

$\therefore m(\widehat{BXY}) = m(\widehat{CYX})$

$\therefore m(\angle C) = m(\angle B)$

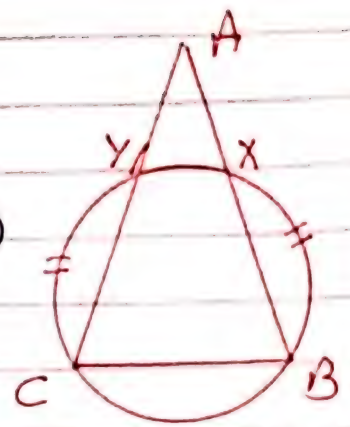
inscribed angles subtended by equal arcs

In  $\triangle ABC$   $\therefore m(\angle B) = m(\angle C)$

$\therefore AB = AC$  — ①

$\therefore m(\widehat{BX}) = m(\widehat{CY}) \therefore BX = CY$  — ②

by subtracting ② from ①  $\therefore AX = AY$



أحمد محمد

$\overline{AB}$  and  $\overline{CD}$  are two equal chords in length in the circle

prove that:  $\triangle ACE$  is an isosceles  $\triangle$

Solution:

$\therefore AB = CD$

$\therefore m(\widehat{ADB}) = m(\widehat{CBD})$

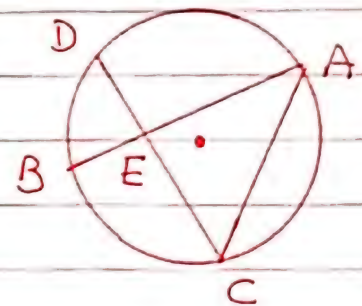
by deleting  $m(\widehat{BD})$  from the two sides

$\therefore m(\widehat{AD}) = m(\widehat{BC})$

$\therefore m(\angle C) = m(\angle A)$  inscribed angles subtended by equal arcs.

In  $\triangle AEC$   $\therefore m(\angle A) = m(\angle C)$

$\therefore \triangle AEC$  is an isosceles triangle





19

prove that: AXEY is a cyclic quad.

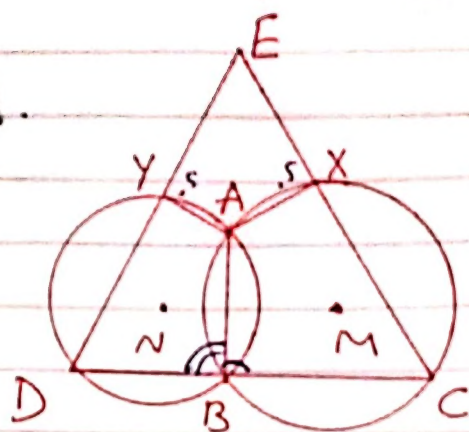
proof:

$\therefore$  AXCB is a cyclic quad.

$\therefore m(\angle AXE)_{\text{exterior}} = m(\angle ABC)$

$\rightarrow \textcircled{1}$

$\therefore$  AB DY is a cyclic quad.



$\therefore m(\angle AYE)_{\text{exterior}} = m(\angle ABD)_{\text{interior}} \rightarrow \textcircled{2}$

by adding  $\textcircled{1}$  and  $\textcircled{2}$

إجمالي

$\therefore m(\angle AXE) + m(\angle AYE) = m(\angle ABC) + m(\angle ABD)$   
 $= 180^\circ$  "B on CD"

and they are opposite

$\therefore$  AXEY is a cyclic quadrilateral.

prove that:  $\textcircled{1}$  AEFD is a cyclic quad.

$\textcircled{2}$   $\overline{EF} \parallel \overline{BC}$

proof:  $\therefore$  ABCD is a cyclic quad.

$\therefore m(\angle BAC) = m(\angle BDC) \rightarrow \textcircled{1}$

drawn on  $\overline{BC}$

$\therefore \overline{AE}$  bisects  $\angle BAC \therefore m(\angle EAF) = \frac{1}{2} m(\angle BAC) \rightarrow \textcircled{2}$

$\therefore \overline{DF}$  bisects  $\angle BDC \therefore m(\angle EDF) = \frac{1}{2} m(\angle BDC) \rightarrow \textcircled{3}$

from  $\textcircled{1}$ ,  $\textcircled{2}$  and  $\textcircled{3}$

$\therefore m(\angle EAF) = m(\angle EDF)$

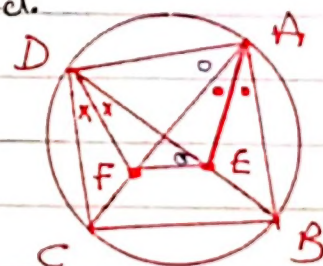
and they are drawn on one side of  $\overline{EF}$

$\therefore$  AEFD is a cyclic quadrilateral.

$\therefore m(\angle DAF) = m(\angle DEF)$  drawn on  $\overline{DF} \rightarrow \textcircled{4}$

$\therefore m(\angle DAC) = m(\angle DBC)$  inscribed sub. by  $\overline{DC} \rightarrow \textcircled{5}$

from  $\textcircled{4}$  and  $\textcircled{5} \therefore m(\angle DEF) = m(\angle DBC) \therefore \overline{EF} \parallel \overline{BC}$





the Circle  $M$  has Circumference  $= 44$  cm.

$\overline{AB}$  is a diameter,  $\overline{BC}$  is tangent at  $B$

$$m\angle C = 60^\circ, \pi = \frac{22}{7}$$

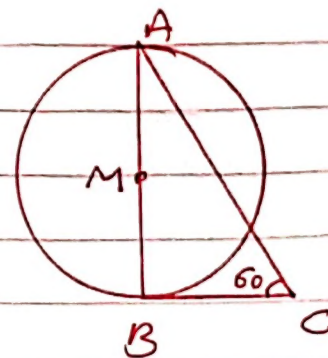
find: the length of  $\overline{BC}$

Solution

$$C = 2\pi r$$

$$\Rightarrow 44 = 2 \times \frac{22}{7} \times r \Rightarrow r = 7 \text{ cm}$$

$$\therefore AB = 14 \text{ cm}$$



$\therefore \overline{AB}$  a diameter and  $\overline{BC}$  is a tangent

$$\therefore m\angle B = 90^\circ$$

$$\tan C = \frac{\text{opp.}}{\text{adj.}} = \frac{AB}{BC}$$

$$\therefore \tan 60 = \frac{14}{BC}$$

$$\therefore BC = \frac{14}{\sqrt{3}} = \frac{14\sqrt{3}}{3} \approx 8.1 \text{ cm}$$

$\overline{BC}$  is a diameter in the Circle  $M$ ,  $\overline{BY}$  is a chord

$H \in \overline{BY}$  such that  $BY = YH$

prove that:  $m\angle YMC = 2m\angle BHC$

proof

In  $\triangle HBC$

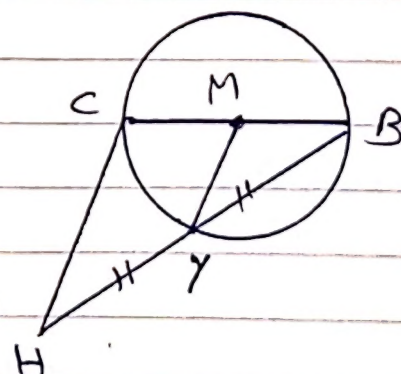
$\therefore M$  is the midpoint of  $\overline{BC}$

and  $Y$  is the midpoint of  $\overline{BH}$

$$\therefore MY = \frac{1}{2} CH \Rightarrow CH = 2MY$$

$$\therefore CH = 2r = BC$$

$$\therefore HC = BC \therefore m\angle H = m\angle B \dots \textcircled{1}$$



$$m\angle YMC \text{ central} = 2m\angle B \text{ inscribed} \dots \textcircled{2}$$

from  $\textcircled{1}$  and  $\textcircled{2}$

$$\therefore m\angle YMC = 2m\angle BHC \quad \#$$



(21)

$ABC$  inscribed triangle in a circle such that  
 $AB > AC$ ,  $D \in \overline{AB}$ ,  $AC = AD$   
 $\overline{AE}$  bisects  $\angle A$  and intersects  $\overline{BC}$  at  $E$  and  
intersects the circle at  $F$   
prove that:  $BDEF$  is a cyclic quadrilateral.

proof:

In  $\triangle ADE, ACE$   $\begin{cases} AD = AC \\ \overline{AE} \text{ is a common side} \\ m(\angle DAE) = m(\angle CAE) \end{cases}$

$$\therefore \triangle ADE \cong \triangle ACE$$

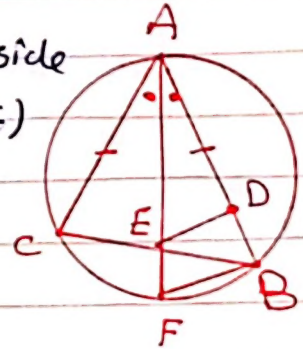
$$\therefore m(\angle ACE) = m(\angle ADE) \dots \textcircled{1}$$

$$\therefore m(\angle ACD) = m(\angle AFD) \dots \textcircled{2}$$

inscribed subtended by  $\widehat{AD}$   
from  $\textcircled{1}$  and  $\textcircled{2}$

$$\therefore m(\angle ADE) \text{ exterior} = m(\angle EFD) \text{ interior}$$

$\therefore BDEF$  is a cyclic quadrilateral.



$ABCD$  is a parallelogram,  $DH = DC$   
prove that  $\textcircled{1} ABHD$  is cyclic quad.

$\textcircled{2} \overline{AD}$  is a tangent of circle passing  
through triangle  $DHC$ .

Solution

$\therefore ABCD$  is a parallelogram

$$\therefore m(\angle C) = m(\angle A) \rightarrow \textcircled{1}$$

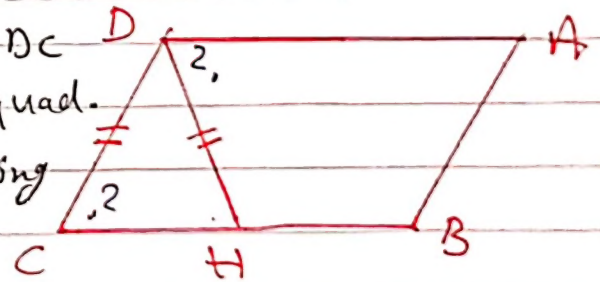
In  $\triangle DHC$ :  $\because DH = DC$

$$\therefore m(\angle C) = m(\angle DHC) \rightarrow \textcircled{2}$$

from  $\textcircled{1}$  and  $\textcircled{2}$

$$\therefore m(\angle DHC) \text{ exterior} = m(\angle A)$$

$\therefore ABHD$  is a cyclic quadrilateral.



$$\therefore \overline{AD} \parallel \overline{BC}$$

$$\therefore m(\angle ADH) = m(\angle DHC) \text{ alternate.}$$

$$\therefore m(\angle C) = m(\angle DHC)$$

$$\therefore m(\angle ADH) = m(\angle C)$$

$\therefore \overline{AD}$  is a tangent.

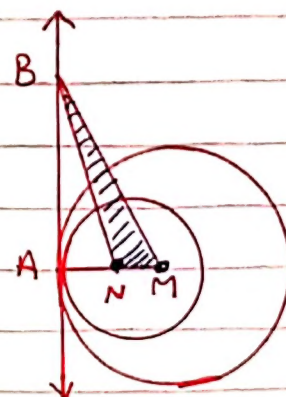
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M and N are two circles whose radii lengths are 10 cm and 6 cm

and touching internally at A,  $\vec{AB}$  is a common tangent at A

If the area of  $\triangle BMN = 24 \text{ cm}^2$  find length of  $\vec{AB}$



Solution:

$\therefore$  the two circles touching internally

$$\therefore MN = r_1 - r_2 = 10 - 6 = 4 \text{ cm}$$

$\therefore \vec{AB}$  is a common tangent

$$\therefore \vec{MN} \perp \vec{AB}$$

$$\text{area of } \triangle BMN = \frac{1}{2} MN \times AB$$

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$$\Rightarrow \frac{1}{2} \times 4 \times AB = 24 \Rightarrow 2AB = 24 \Rightarrow AB = 12 \text{ cm}$$

ABCD is a quadrilateral in which,  $AB = AD$ ,

$m(\angle ABD) = 30^\circ$  and  $m(\angle C) = 60^\circ$

prove that: ABCD is a cyclic quadrilateral.

Solution:

In  $\triangle ABD$

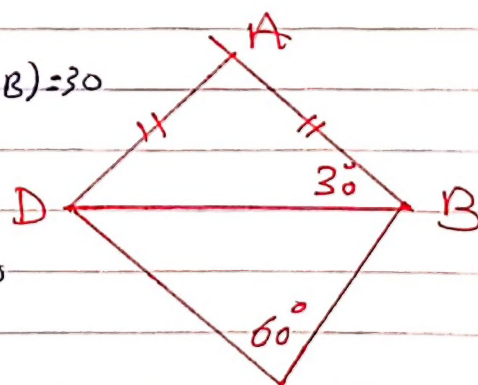
$$\because AB = AD \therefore m(\angle ABD) = m(\angle ADB) = 30^\circ$$

$$\therefore m(\angle A) = 180 - (30 + 30) = 120$$

$$\therefore m(\angle A) + m(\angle C) = 120 + 60 = 180$$

and they are opposite

$\therefore$  ABCD is a cyclic quadrilateral. C



مع أطيب الأمانات بالإنجاز وليتقوه بآه

أحمد عمر ١٧ / ٥ / ٢٠٢١